

Tečna a normála

$$f(x) = \frac{1 + \cos x}{1 + \sin x}$$

$$T = \left[ \frac{\pi}{4}, 1 \right]$$

$$I) f\left(\frac{\pi}{4}\right) = \frac{1 + \cos \frac{\pi}{4}}{1 + \sin \frac{\pi}{4}} = \frac{1 + \frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}} = 1$$

$$II) f'(x) = \frac{-\sin x (1 + \sin x) - (1 + \cos x) \cdot \cos x}{(1 + \sin x)^2}$$

$$III) \underline{f'\left(\frac{\pi}{4}\right)} = \frac{-\sin \frac{\pi}{4} (1 + \sin \frac{\pi}{4}) - (1 + \cos \frac{\pi}{4}) \cdot \cos \frac{\pi}{4}}{(1 + \sin \frac{\pi}{4})^2} = \frac{-\frac{\sqrt{2}}{2} \left(1 + \frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2} \left(1 + \frac{\sqrt{2}}{2}\right)}{\left(1 + \frac{\sqrt{2}}{2}\right)^2}$$

$$= \frac{-\frac{\sqrt{2}}{2} - \frac{2}{4} - \frac{\sqrt{2}}{2} - \frac{2}{4}}{1 + \frac{2\sqrt{2}}{2} + \frac{2}{4}} = \frac{-\sqrt{2} - 1}{1 + \sqrt{2} + \frac{1}{2}} = \frac{-\sqrt{2} - 1}{\frac{2 + 2\sqrt{2} + 1}{2}} = \frac{-\sqrt{2} - 1}{1} \cdot \frac{2}{2 + 2\sqrt{2} + 1} =$$

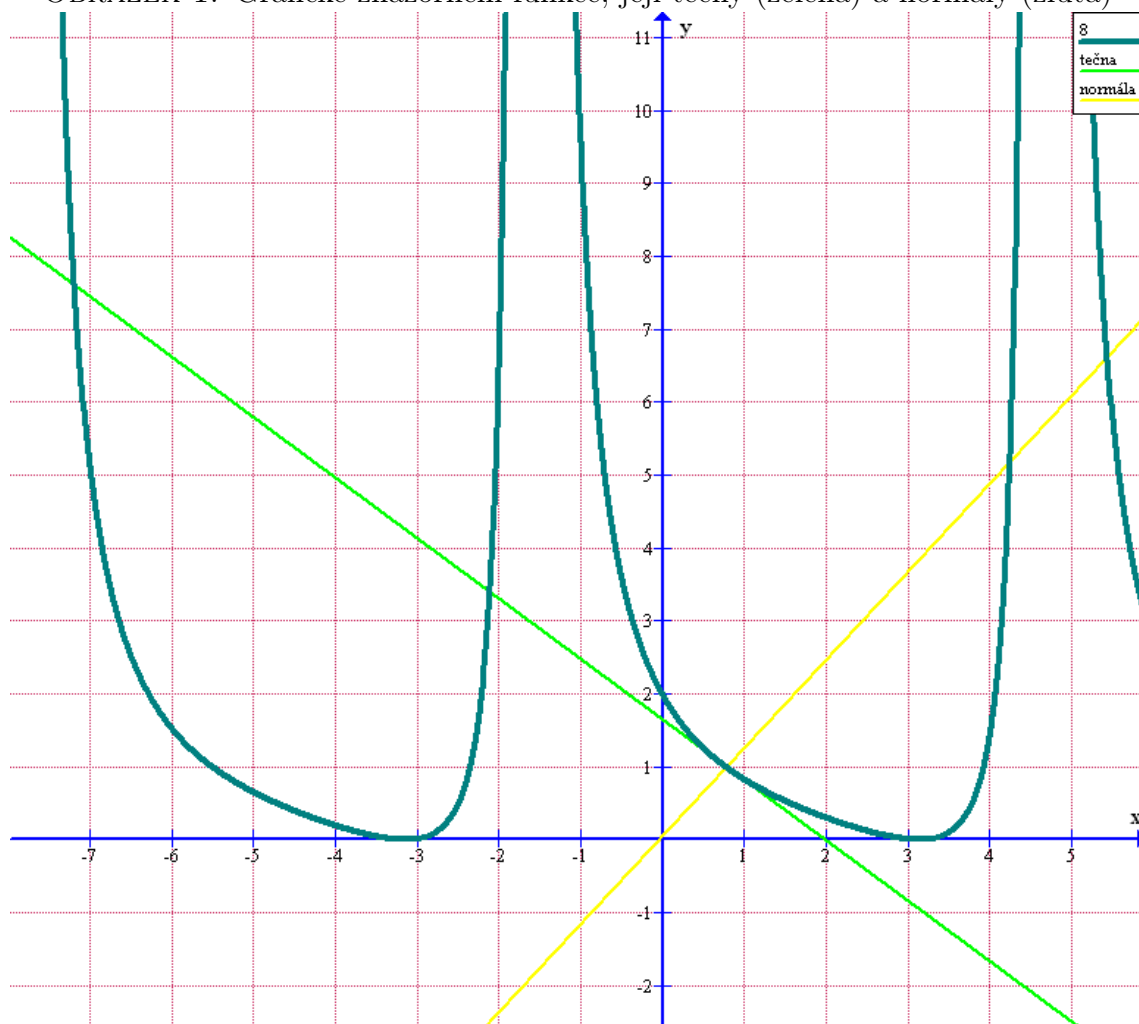
$$= -\sqrt{2} - 1 \cdot \frac{2}{2\sqrt{2} + 3} = \underline{\underline{\frac{-2\sqrt{2} - 2}{2\sqrt{2} + 3}}}$$

$$t: \underline{\underline{y - 1 = \frac{-2\sqrt{2} - 2}{2\sqrt{2} + 3} \left(x - \frac{\pi}{4}\right)}}$$

$$n: \underline{\underline{y - 1 = \frac{2\sqrt{2} + 3}{2(\sqrt{2} + 1)} \left(x - \frac{\pi}{4}\right)}}$$

moc ošklivý, není ostrý (ale vadší se procvičte)

OBRÁZEK 1. Grafické znázornění funkce, její tečny (zelená) a normály (žlutá)



Zdroj: program Graph