

$$1) \int \cos 5x dx = \left| \begin{array}{l} 5x = t \\ 5dx = dt \\ dx = \frac{dt}{5} \end{array} \right| = \int \cos t \cdot \frac{dt}{5} = \frac{1}{5} \sin t + C$$

Substituce zpět: $\frac{1}{5} \sin 5x + C$

$$2) \int \sin(8-9x) dx = \left| \begin{array}{l} 8-9x = t \\ -9dx = dt \\ dx = \frac{-dt}{9} \end{array} \right| = \int \sin t \cdot \left(\frac{-dt}{9}\right) = +\frac{1}{9} \cos t + C$$

Substituce zpět: $\frac{1}{9} \cos(8-9x) + C$

$$3) \int 3x \cdot e^{x^2} dx = \left| \begin{array}{l} x^2 = t \\ 2x dx = dt \\ x dx = \frac{dt}{2} \end{array} \right| = \frac{3}{2} \int e^t dt = \frac{3}{2} e^t + C$$

Substituce zpět: $\frac{3}{2} e^{x^2} + C$

$$4) \int \cot x dx = \int \frac{\cos x}{\sin x} dx = \left| \begin{array}{l} \sin x = t \\ \cos x dx = dt \end{array} \right| = \int \frac{dt}{t} = \ln |t| + C$$

Substituce zpět: $\ln |\sin x| + C$

$$5) \int \frac{1}{x \cdot \ln^4 x} dx = \int \left(\frac{1}{x}\right) \cdot \left(\frac{1}{\ln^4 x}\right) dx = \left| \begin{array}{l} \ln x = t \\ \frac{1}{x} dx = dt \end{array} \right| = \int \frac{1}{t^4} \cdot dt = \int t^{-4} dt =$$

$$= \frac{t^{-3}}{-3} + C = \frac{1}{-3t^3} + C$$

Sg: $\frac{1}{-3 \ln x} + C$

$$6) \int \frac{x}{(3+x^2)^2} dx = \left| \begin{array}{l} 3+x^2 = t \\ 2x dx = dt \\ x dx = \frac{dt}{2} \end{array} \right| = \int \frac{1}{t^2} \cdot \frac{dt}{2} = \frac{1}{2} \int t^{-2} dt = \frac{1}{2} \cdot \frac{t^{-1}}{-1} + C =$$

$$= \frac{1}{-2t} + C$$

$$\text{sg: } \frac{-1}{2(3+x^2)} + C$$

$$7) \int \frac{1}{\sqrt{2+3x}} dx = \left| \begin{array}{l} \sqrt{2+3x} = t \\ 2+3x = t^2 \\ 3dx = 2t dt \\ dx = \frac{2t}{3} dt \end{array} \right| = \int \frac{1}{t} \cdot \frac{2t}{3} dt = \frac{2}{3} \int dt = \frac{2}{3} t + C$$

$$\text{Substitution sg: } \frac{2}{3} \sqrt{2+3x} + C$$

$$8) \int x^4 e^{2x^5+5} dx = \left| \begin{array}{l} 2x^5+5 = t \\ 10x^4 dx = dt \\ x^4 dx = \frac{dt}{10} \end{array} \right| = \int e^t \cdot \frac{dt}{10} = \frac{1}{10} e^t + C$$

$$\text{sg: } \frac{1}{10} e^{2x^5+5} + C$$

$$9) \int \frac{1}{x^2} \cdot \cos \frac{1}{x} dx = \left| \begin{array}{l} \frac{1}{x} = t \\ -\frac{1}{x^2} dx = dt \end{array} \right| = -\int \cos t dt = -\sin t + C \quad \text{sg: } -\sin \frac{1}{x} + C$$

$$10) \int x \cdot \sqrt{x+1} dx = \left| \begin{array}{l} \sqrt{x+1} = t \\ x+1 = t^2 \\ dx = 2t dt \\ x = t^2 - 1 \end{array} \right| = \int (t^2 - 1) \cdot t \cdot 2t dt = \int (2t^4 - 2t^2) dt =$$

$$= 2 \int t^4 dt - 2 \int t^2 dt = 2 \frac{t^5}{5} - 2 \frac{t^3}{3} + C$$

$$\text{Substitution sg: } \frac{2(\sqrt{x+1})^5}{5} - \frac{2(\sqrt{x+1})^3}{3} + C$$

$$11) \int 4x^3 \sin x^4 dx = \left| \begin{array}{l} x^4 = t \\ 4x^3 dx = dt \end{array} \right| = \int \sin t \cdot dt = -\cos t + C$$

Substituce zmiť $-\cos x^4 + C$

$$12) \int \sin^2 x dx = \int \frac{1 - \cos 2x}{2} dx = \left| \begin{array}{l} 2x = t \\ 2dx = dt \\ dx = \frac{dt}{2} \end{array} \right| = \int \frac{1 - \cos t}{2} \cdot \frac{dt}{2} =$$

$$= \frac{1}{4} \int (1 - \cos t) dt = \frac{1}{4} (t - \sin t) + C$$

Tento príklad lze vřít
i metodou per partes
- viz př. 14.

$$\text{ř: } \frac{2x}{4} - \frac{\sin 2x}{4} + C = \frac{2x - (2 \cdot \sin x \cdot \cos x)}{4} + C = \frac{x - \sin x \cdot \cos x}{2} + C$$

$$13) \int (1 + e^{2x})^3 \cdot e^{2x} dx = \left| \begin{array}{l} 1 + e^{2x} = t \\ 2e^{2x} = dt \\ e^{2x} = \frac{dt}{2} \end{array} \right| = \int t^3 \cdot \frac{dt}{2} = \frac{1}{2} \int t^3 dt =$$

$$= \frac{1}{2} \cdot \frac{t^4}{4} + C = \frac{t^4}{8} + C$$

$$\text{ř: } \frac{(1 + e^{2x})^4}{8} + C$$

$$14) \int \frac{\sin 2x}{\sqrt{1 + \sin^2 x}} dx = \left| \begin{array}{l} \sqrt{1 + \sin^2 x} = t \\ 1 + \sin^2 x = t^2 \\ 2 \sin x \cdot \cos x dx = 2t dt \\ \sin 2x dx = 2t dt \end{array} \right| = \int \frac{1}{t} \cdot 2t dt = 2 \int dt = 2t + C$$

$$\text{ř: } 2 \sqrt{1 + \sin^2 x} + C$$

$$15) \int \frac{\cos x - \sin x}{\sin x + \cos x} dx = \left| \begin{array}{l} \sin x + \cos x = t \\ (\cos x - \sin x) dx = dt \end{array} \right| = \int \frac{dt}{t} = \ln |t| + C$$

Substituce zmiť: $\ln |\sin x + \cos x| + C$

$$16) \int \frac{\arcsin^3 x - 3x}{\sqrt{1-x^2}} dx = \underbrace{\int \frac{(\arcsin x)^3}{\sqrt{1-x^2}} dx}_{I_1} - \underbrace{3 \int \frac{x}{\sqrt{1-x^2}} dx}_{I_2}$$

$$I_1: \left| \begin{array}{l} \arcsin x = t \\ \frac{1}{\sqrt{1-x^2}} dx = dt \end{array} \right| = \int t^3 \cdot dt = \frac{t^4}{4} + C \quad \text{sg: } \frac{1}{4} \arcsin^4 x + C$$

$$I_2: \left| \begin{array}{l} \sqrt{1-x^2} = s \\ 1-x^2 = s^2 \\ -2x dx = 2s ds \\ x dx = \frac{-2s ds}{2} \end{array} \right| = 3 \int \frac{-s ds}{s} = -3 \int ds = -3s + C \quad \text{sg: } -3\sqrt{1-x^2} + C$$

$$I_1 + I_2 = \frac{1}{4} \arcsin^4 x - 3\sqrt{1-x^2} + C$$

$$17) \int \frac{1}{x} (3 + 2x^2 \cdot e^{x^2+1}) dx =$$

tento príklad je plánovaný pro
Lesnickou fakultu

$$= \underbrace{3 \int \frac{1}{x} dx}_{I_1} + \underbrace{2 \int x \cdot e^{x^2+1} dx}_{I_2}$$

$$I_1: 3 \ln|x| + C$$

$$I_2: \left| \begin{array}{l} x^2+1 = t \\ 2x dx = dt \end{array} \right| = \int e^t dt = e^t + C \quad \text{sg: } e^{x^2+1} + C$$

$$I_1 + I_2 = 3 \ln|x| + e^{x^2+1} + C$$

$$18) \int \ln(1+\sqrt{x}) dx = \left| \begin{array}{l} 1+\sqrt{x} = t \\ \sqrt{x} = t-1 \\ x = (t-1)^2 \\ s. dx = 2(t-1) dt \end{array} \right| = \int \ln t \cdot 2(t-1) dt = \left| \begin{array}{l} u' = 2(t-1) \quad v = \ln t \\ u = (t-1)^2 \quad v' = \frac{1}{t} \end{array} \right| =$$

$$= \underbrace{(t-1)^2 \cdot \ln t}_A - \int (t-1)^2 \cdot \frac{1}{t} dt = A - \int (t^2 - 2t + 1) \cdot \frac{1}{t} dt = A - \int t dx + \int 2 dx - \int \frac{1}{t} dt = \dots$$

$$\text{sg: } x \ln(1+\sqrt{x}) - \frac{(1+\sqrt{x})^2}{2} + 2(1+\sqrt{x}) - \ln|1+\sqrt{x}| + C$$

$$19) \int \frac{\arccos^2 x + x}{\sqrt{1-x^2}} dx = \underbrace{\int \frac{(\arccos x)^2}{\sqrt{1-x^2}} dx}_{I_1} + \underbrace{\int \frac{x}{\sqrt{1-x^2}} dx}_{I_2}$$

$$I_1: \left| \begin{array}{l} \arccos x = t \\ -\frac{1}{\sqrt{1-x^2}} dx = dt \end{array} \right| = -\int t^2 dt = -\frac{t^3}{3} + C \quad \text{sg: } \underline{\underline{-\frac{(\arccos x)^3}{3} + C}}$$

$$I_2: \left| \begin{array}{l} \sqrt{1-x^2} = t \\ 1-x^2 = t^2 \\ -2x dx = 2t dt \\ x dx = -t dt \end{array} \right| = \int \frac{-t dt}{t} = -\int dt = -t + C \quad \text{sg: } \underline{\underline{-\sqrt{1-x^2} + C}}$$

$$I_1 + I_2 = \underline{\underline{-\frac{1}{3} \arccos^3 x - \sqrt{1-x^2} + C}}$$

$$20) \int \frac{dx}{\cos^2 x \sqrt{\lg x - 1}} = \left| \begin{array}{l} \sqrt{\lg x - 1} = t \\ \lg x - 1 = t^2 \\ \frac{1}{\cos^2 x} dx = 2t dt \end{array} \right| = \int \frac{1}{t} \cdot 2t dt = \int 2 dt = 2t + C$$

$$\text{Substitue mit: } \underline{\underline{2\sqrt{\lg x - 1} + C}}$$

$$21) \int e^{-x} dx = \left| \begin{array}{l} -x = t \\ -dx = dt \\ dx = -dt \end{array} \right| = - \int e^t dt = -e^t + C \quad \text{sg: } \underline{-e^{-x} + C}$$

$$22) \int x^4 e^{4x^5+7} dx = \left| \begin{array}{l} 4x^5+7 = t \\ 20x^4 dx = dt \\ x^4 dx = \frac{dt}{20} \end{array} \right| = \int e^t \cdot \frac{1}{20} dt = \frac{1}{20} e^t + C$$

Substitucija got: $\frac{1}{20} e^{4x^5+7} + C$

$$23) \int \frac{10^{\cot x}}{\sin^2 x} dx = \left| \begin{array}{l} \cot x = t \\ -\frac{1}{\sin^2 x} dx = dt \end{array} \right| = - \int 10^t \cdot dt = - \frac{10^t}{\ln 10} + C$$

Substitucija got: $-\frac{10^{\cot x}}{\ln 10} + C$

$$24) \int \frac{dx}{x \cdot \ln^7 x} = \left| \begin{array}{l} \ln x = t \\ \frac{1}{x} dx = dt \end{array} \right| = \int \frac{1}{t^7} dt = \int t^{-7} dt = \frac{t^{-6}}{-6} + C$$

S.g.: $-\frac{1}{6 (\ln x)^6} + C$

$$25) \int \frac{e^{2x} dx}{\sqrt{e^x - 1}} = \left| \begin{array}{l} \sqrt{e^x - 1} = t \\ e^x - 1 = t^2 \\ e^x dx = 2t dt \end{array} \right| = \int \frac{(t^2 + 1) \cdot 2t}{t} dt = 2 \int (t^2 + 1) dt =$$

$$= 2 \left(\frac{t^3}{3} + t \right) + C = 2t \left(\frac{t^2}{3} + 1 \right) + C$$

sg: $2 \sqrt{e^x - 1} \left(\frac{e^x - 1 + 3}{3} \right) + C = 2 \sqrt{e^x - 1} \cdot \left(\frac{e^x + 2}{3} \right) + C$

$$26) \int \frac{dx}{\sin^2 3x} = \left| \begin{array}{l} 3x = t \\ 3dx = dt \end{array} \right| = \int \frac{1}{\sin^2 t} \cdot \frac{dt}{3} = \frac{1}{3} \cdot (-\cotg t) + C$$

$$\text{sg: } \underline{\underline{\frac{-\cotg 3x}{3} + C}}$$

$$27) \int \cos \frac{2x-3}{5} dx = \left| \begin{array}{l} \frac{2x-3}{5} = t \\ \frac{2x}{5} - \frac{3}{5} = t \\ \frac{2}{5} dx = dt \\ dx = \frac{5}{2} dt \end{array} \right| = \int \cos t \cdot \frac{5}{2} dt = \frac{5}{2} \sin t + C$$

$$\text{sg: } \underline{\underline{\frac{5}{2} \sin \frac{2x-3}{5} + C}}$$

$$28) \int x^3 \cdot \sin x^4 dx = \left| \begin{array}{l} x^4 = t \\ 4x^3 dx = dt \end{array} \right| = \int \sin t \cdot \frac{1}{4} dt = \frac{1}{4} (-\cos t) + C$$

$$\text{sg: } \underline{\underline{\frac{-\cos x^4}{4} + C}}$$

$$29) \int \frac{dx}{1-x} = \left| \begin{array}{l} 1-x = t \\ -dx = dt \end{array} \right| = -\int \frac{dt}{t} = -\ln|t| + C \quad \text{sg: } \underline{\underline{-\ln|1-x| + C}}$$

$$30) \int \frac{e^x dx}{2+e^x} = \left| \begin{array}{l} 2+e^x = t \\ e^x dx = dt \end{array} \right| = \int \frac{dt}{t} = \ln|t| + C \Rightarrow \underline{\underline{\ln|2+e^x| + C}}$$

$$31) \int \frac{x^3}{x^2-1} dx = \left| \begin{array}{l} x^2-1 = t \\ 2x dx = dt \\ x dx = \frac{dt}{2} \end{array} \right| = \int \frac{t+1}{t} \cdot \frac{dt}{2} = \frac{1}{2} \int \left(1 + \frac{1}{t}\right) dt =$$

$$= \frac{1}{2} (t + \ln|t|) + C = \underline{\underline{\frac{x^2-1}{2} + \frac{\ln|x^2-1|}{2} + C}}$$

$$32) \int \frac{x dx}{\sqrt{(x^2-1)^3}} = \left| \begin{array}{l} x^2-1=t \\ 2x dx = dt \\ x dx = \frac{dt}{2} \end{array} \right| = \frac{1}{2} \int \frac{dt}{\sqrt{t^3}} = \frac{1}{2} \int \frac{dt}{t^{\frac{3}{2}}} = \frac{1}{2} \int t^{-\frac{3}{2}} dt =$$

$$= \frac{1}{2} \cdot \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + C = \frac{1}{2} \cdot \frac{1}{\sqrt{t}} \cdot (-2) + C$$

Substitua $\frac{-1}{\sqrt{x^2-1}} + C$

$$33) \int \frac{x^2 dx}{(1-x^3)^2} = \left| \begin{array}{l} 1-x^3=t \\ -3x^2 dx = dt \\ x^2 dx = \frac{-dt}{3} \end{array} \right| = \int \frac{1}{t^2} \cdot \left(-\frac{1}{3}\right) dt = -\frac{1}{3} \int t^{-2} dt =$$

$$= -\frac{1}{3} \cdot \frac{t^{-1}}{-1} + C = \frac{1}{3t} + C \quad \text{sg: } \frac{1}{3(1-x^3)} + C$$

$$34) \int \frac{x^3 dx}{(2+3x^2)^3} = \left| \begin{array}{l} 2+3x^2=t \\ \hookrightarrow x^2 = \frac{t-2}{3} \\ 6x dx = dt \\ x dx = \frac{1}{6} dt \end{array} \right| = \int \frac{\frac{t-2}{3}}{t^3} \cdot \frac{1}{6} dt = \frac{1}{6} \int \frac{t-2}{t^3} \cdot \frac{dt}{t^3} =$$

$$= \frac{1}{6} \int \frac{t-2}{t^3} dt = \frac{1}{18} \int \frac{t-2}{t^3} dt = \frac{1}{18} \int (t^{-2} - 2t^{-3}) dt = \frac{1}{18} \cdot \frac{t^{-1}}{-1} - \frac{2}{18} \cdot \frac{t^{-2}}{-2} + C =$$

$$= \frac{-1}{18t} + \frac{1}{18t^2} + C = \frac{-1}{18(2+3x^2)} + \frac{1}{18(2+3x^2)^2} + C$$

$$35) \int \frac{x dx}{\sqrt{1-x}} = \left| \begin{array}{l} \sqrt{1-x}=t \\ 1-x=t^2 \\ x=1-t^2 \\ -dx = 2t dt \end{array} \right| = \int \frac{1-t^2}{t} (-2t dt) = -2 \int (1-t^2) dt =$$

$$= -2 \left(t - \frac{t^3}{3} \right) + C = \frac{2t^3}{3} - 2t + C = 2t \left(\frac{t^2}{3} - 1 \right) + C$$

sg: $2\sqrt{1-x} \cdot \left(\frac{1-x}{3} - 1 \right) + C = 2\sqrt{1-x} \cdot \left(\frac{1-x-3}{3} \right) + C = \frac{2}{3}\sqrt{1-x} \cdot (-x-2) + C$

$$36) \int \frac{1}{4+x^2} dx = \left| \begin{array}{lcl} x^2 & = & 4t^2 \\ x & = & 2t \\ dx & = & 2 dt \end{array} \right| = \int \frac{1}{4+4t^2} \cdot 2 dt = \int \frac{1}{4 \cdot (1+t^2)} \cdot 2 dt = \frac{1}{2} \cdot \int \frac{1}{1+t^2} dt = \frac{1}{2} \cdot \operatorname{arctg} t + C =$$

$$\frac{1}{2} \cdot \operatorname{arctg} \left(\frac{x}{2} \right) + C$$

substituce zpět: $\frac{1}{2} \cdot \operatorname{arctg} \left(\frac{x}{2} \right) + C$

$$37) \int \frac{\cos x}{4+\sin^2 x} dx = \left| \begin{array}{lcl} (\sin x)^2 & = & 4t^2 \\ \sin x & = & 2t \\ \cos dx & = & 2 dt \end{array} \right| = \int \frac{2 dt}{4+4t^2} = \int \frac{2 dt}{4 \cdot (1+t^2)} = \frac{1}{2} \cdot \int \frac{dt}{1+t^2} = \frac{1}{2} \cdot \operatorname{arctg} t + C$$

substituce zpět: $\frac{1}{2} \cdot \operatorname{arctg} \left(\frac{\sin x}{2} \right) + C$

$$38) \int \frac{e^{2x}}{\sqrt{e^x-1}} dx = \int \frac{e^x \cdot e^x}{\sqrt{e^x-1}} dx = \left| \begin{array}{lcl} \sqrt{e^x-1} & = & t \\ e^x-1 & = & t^2 \\ e^x & = & t^2+1 \\ e^x dx & = & 2t dt \end{array} \right| = \int \frac{t^2+1}{t} \cdot 2t dt = 2 \cdot \int (t^2+1) dt = 2 \cdot \left(\frac{t^3}{3} + t \right) + C$$

$$= 2 \cdot \left(\frac{(\sqrt{e^x-1})^3}{3} + \sqrt{e^x-1} \right) + C$$

substituce zpět: $2 \cdot \left(\frac{\sqrt{e^x-1} \cdot (e^x-1)}{3} + \sqrt{e^x-1} \right) + C = 2 \cdot \sqrt{e^x-1} \cdot \left(\frac{e^x-1}{3} + 1 \right) + C$

$$39) \int \frac{\ln x}{x \cdot \sqrt{1+\ln x}} dx = \left| \begin{array}{lcl} \sqrt{1+\ln x} & = & t \\ 1+\ln x & = & t^2 \\ \frac{1}{x} dx & = & 2t dt \end{array} \right| = \int \frac{t^2-1}{t} \cdot 2t dt = 2 \cdot \int (t^2-1) dt = 2 \cdot \left(\frac{t^3}{3} - t \right) + C$$

substituce zpět: $2 \cdot \left(\frac{(1+\ln x) \cdot \sqrt{1+\ln x}}{3} - \sqrt{1+\ln x} \right) + C = 2 \cdot \sqrt{1+\ln x} \cdot \left(\frac{1+\ln x}{3} - 1 \right) + C =$

$2 \cdot \sqrt{1+\ln x} \cdot \left(\frac{\ln x - 2}{3} \right) + C$