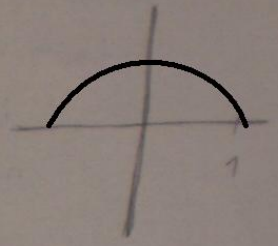


APLIKACE URČITÉHO INTEGRÁLU

Délka křivky:

$$y = \sqrt{4-x^2}$$

$$x \in \langle 0; 1 \rangle$$



$$l = \int_0^1 \sqrt{1 + \frac{x}{4-x^2}} dx =$$

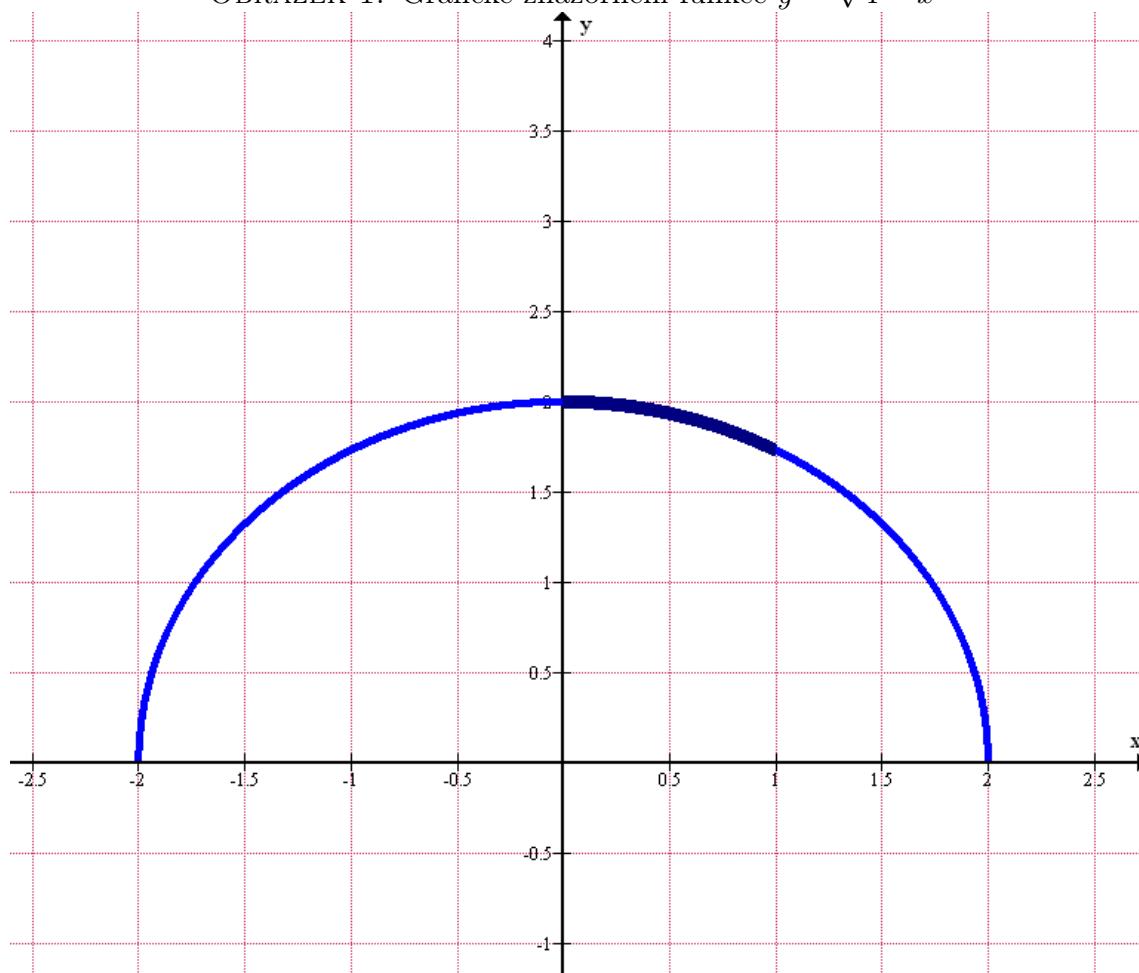
$$(f'(x))^2 = (\sqrt{4-x^2})' = \frac{1}{2\sqrt{4-x^2}} \cdot (-2x) = \left(\frac{-x}{\sqrt{4-x^2}}\right)^2 = \frac{(-x)^2}{(\sqrt{4-x^2})^2} = \frac{x}{4-x^2}$$

$$= \int_0^1 \sqrt{\frac{4-x^2+x^2}{4-x^2}} dx = \int_0^1 \frac{\sqrt{4}}{\sqrt{4-x^2}} dx = \left. \begin{array}{l} x^2 = 4t^2 \text{ , změna mezí} \\ x = 2t \quad | x=0 \dots t=0 \\ dx = 2dt \quad | x=1 \dots t=\frac{1}{2} \end{array} \right| =$$

$$= \int_0^{\frac{1}{2}} \frac{2}{\sqrt{4-4t^2}} 2dt = 4 \int_0^{\frac{1}{2}} \frac{dt}{2\sqrt{1-t^2}} = 2 \int_0^{\frac{1}{2}} \frac{dt}{\sqrt{1-t^2}} = 2 \left[\arcsin t \right]_0^{\frac{1}{2}} =$$

$$= 2 \left(\arcsin \frac{1}{2} - \arcsin 0 \right) = 2 \left(\frac{\pi}{6} - 0 \right) = \underline{\underline{\frac{\pi}{3}}} \text{ délkových jednotek}$$

OBRÁZEK 1. Grafické znázornění funkce $y = \sqrt{4 - x^2}$



Zdroj: program Graph