Př 1) $\int \left(3\sin x + \frac{2}{1+x^2} - \frac{5}{x^6} + 7e^x\right)dx =$

$= 3\int \sin x\, dx + 2\int \frac{dx}{1+x^2} - 5\int x^{-6}\, dx + 7\int e^x\, dx =$

$= -3\cos x + 2\,\mathrm{arctg}\, x - 5\frac{x^{-5}}{(-5)} + 7e^x + C =$

$= 2\,\mathrm{arctg}\, x - 3\cos x + \frac{1}{x^5} + 7e^x + C$

Př 2) $\int \frac{1-x^3}{x^4}\, dx = \int \left(\frac{1}{x^4} - \frac{x^3}{x^4}\right)dx = \int x^{-4}\, dx - \int \frac{1}{x}\, dx =$

$= \frac{x^{-3}}{-3} - \ln|x| + C$

Př 3) $\int \frac{(x^2+1)^2}{x^3}\, dx = \int \frac{x^4+2x^2+1}{x^3}\, dx = \int \frac{x^4}{x^3}\, dx + 2\int \frac{x^2}{x^3}\, dx + \int \frac{1}{x^3}\, dx =$

$= \int x\, dx + 2\int \frac{1}{x}\, dx + \int x^{-3}\, dx = \frac{x^2}{2} + 2\ln|x| + \frac{x^{-2}}{-2} + C =$

$= \frac{x^2}{2} + 2\ln|x| - \frac{1}{2x^2} + C$

Př 4) $\int e^x \cdot 3^x\, dx = \int (3e)^x\, dx = \frac{(3e)^x}{\ln 3e} + C$ (3e je konstanta)

Př 5) $\displaystyle\int \underbrace{\frac{x^4+x^3-x^2+x+4}{x^2+1}} dx =$

$$(x^4+x^3-x^2+x+4):(x^2+1) = x^2+x-2$$

$$\frac{-x^4 \quad -x^2}{x^3-2x^2+x+4}$$

nejvyšší dělím nejvyšším, dokud není v prvním členu menší člen než v děliteli

$$\int\left(x^2+x-2+\frac{\overset{\text{zbytek}}{6}}{\underset{\text{dělitel}}{x^2+1}}\right)dx = \frac{x^3}{3}+\frac{x^2}{2}-2x+6\,\mathrm{arctg}\,x+C$$

Př 6) $\displaystyle\int 10^x\left(1+\frac{10^{-x}}{\cos^2 x}\right)dx = \int\left(10^x+\frac{10^x\cdot 10^{-x}}{\cos^2 x}\right)dx = \frac{10^x}{\ln 10}+\int\frac{1}{\cos^2 x}\,dx$

$$= \frac{10^x}{\ln 10}+\mathrm{tg}\,x+C$$

Př 7) $\displaystyle\int\frac{x^2}{1+x^2}\,dx = \int\frac{x^2+1-1}{1+x^2}\,dx =$

$$= \int\frac{1+x^2}{1+x^2}\,dx-\int\frac{1}{1+x^2}\,dx \Longrightarrow x-\mathrm{arctg}\,x+C$$

$$\Longrightarrow x+\mathrm{arccotg}\,x+C$$

Př 8) $\int \sqrt[3]{x} = \int x^{\frac{1}{3}} dx = \dfrac{x^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{3}{4}\sqrt[3]{x^4} + C$

Př 9) $\int 3e\pi \, dx = 3e\pi x + C$

Př 10) $\int \dfrac{1}{\sqrt[7]{x}} dx = \int x^{-\frac{1}{7}} dx = \dfrac{x^{\frac{6}{7}}}{\frac{6}{7}} = \frac{7}{6}\sqrt[7]{x^6} + C$

Př 11) $\int \dfrac{2x}{3\sqrt{x}} dx = \frac{2}{3}\int \sqrt{x}\, dx = \frac{2}{3}\int x^{\frac{1}{2}} dx = \frac{2}{3} \cdot \dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + C =$

$= \frac{2}{3} \cdot \frac{2}{3} \sqrt[2]{x^3} + C = \frac{4}{9} x\sqrt{x} + C$

zkouška - derivace výsledku

$\hookrightarrow \left(\frac{4}{9} \underset{x^{\frac{3}{2}}}{x\sqrt{x}} + C\right)' = \frac{\cancel{4}}{\cancel{9}} \cdot \frac{\cancel{3}}{\cancel{2}} \cdot x^{\frac{1}{2}} = \frac{2}{3}\sqrt{x} + 0 = \frac{2\sqrt{x}}{3} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \boxed{\frac{2x}{3\sqrt{x}}}$

Př 12) $\int \dfrac{dx}{e^{5x}} = \int \left(\frac{1}{e^5}\right)^x dx = \int \left(e^{-5}\right)^x dx = \dfrac{e^{-5x}}{\ln(e^{-5})} + C =$

$= \dfrac{e^{-5x}}{-5 \cdot \ln e} + C = \dfrac{e^{-5x}}{-5} + C$

Př 13) $\int \left(2x^3 - \frac{x}{2} + \frac{1}{x}\right) dx = 2\int x^3 dx - \frac{1}{2}\int x\, dx + \int \frac{1}{x} dx =$

$= 2\frac{x^4}{4} - \frac{1}{2}\cdot\frac{x^2}{2} + \ln|x| + C = \frac{x^4}{2} - \frac{x^2}{4} + \ln|x| + C$

Př 14) $\int\left(\frac{1}{\sqrt[4]{x^3}}+\frac{1}{\sqrt{x}}\right)dx=\int\frac{1}{x^{\frac{3}{4}}}dx+\int\frac{1}{x^{\frac{1}{2}}}dx=\int x^{-\frac{3}{4}}dx+\int x^{-\frac{1}{2}}dx=$

$=\frac{x^{\frac{1}{4}}}{\frac{1}{4}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C=4\sqrt[4]{x}+2\sqrt{x}+C$

Př 15) $\int(2x^3-5x^2+8x-3)dx=2\int x^3dx-5\int x^2dx+8\int x\,dx-3\int dx=$

$=2\frac{x^4}{4}-5\frac{x^3}{3}+8\frac{x^2}{2}-3x+C=\frac{x^4}{2}-\frac{5x^3}{3}+4x^2-3x+C$

Př 16) $\int\frac{(2x^2+1)^2}{x^3}dx=\int\frac{4x^4+4x^2+1}{x^3}dx=\int\frac{4x^4}{x^3}dx+\int\frac{4x^2}{x^3}dx+\int\frac{1}{x^3}dx=$

$=4\int x\,dx+4\int\frac{1}{x}dx+\int x^{-3}dx=4\frac{x^2}{2}+4\ln|x|+\frac{x^{-2}}{-2}+C=$

$=2x^2+4\ln|x|-\frac{1}{2x^2}+C$

Př 17) $\int\left(\frac{2}{\cos^2 x}-3\cos x+1\right)dx=2\int\frac{1}{\cos^2 x}dx-3\int\cos x\,dx+\int dx=$

$=2\,\mathrm{tg}\,x-3\sin x+x+C$

Př 18) $\int\left(5\sin x+\frac{1}{\sin^2 x}-2\right)dx=-5\cos x-\mathrm{cotg}\,x-2x+C$

Př 19) $\int e^{3x}\cdot 2^x\,dx = \int e^3\cdot e^x\cdot 2^x\,dx = e^3\int (2e)^x\,dx = e^3\cdot\frac{(2e)^x}{\ln 2e}+C =$

$= \frac{2^x\cdot e^x}{\ln 2e}+C$

Př 20) $\int 4^x\cdot\left(1-\frac{4^{-x}}{1+x^2}\right)dx = \int\left(4^x-\frac{1}{1+x^2}\right)dx = \int 4^x\,dx-\int\frac{1}{1+x^2}\,dx =$

$= \frac{4^x}{\ln 4}-\operatorname{arctg} x + C$

Př 21) $\int\frac{1}{\cos 2x-\cos^2 x}\,dx = \int\frac{\sin^2 x+\cos^2 x}{\cancel{\cos^2 x}-\sin^2 x-\cancel{\cos^2 x}}\,dx = -\int\frac{\sin^2 x}{\cos^2 x}\,dx$

$-\int\frac{\boxed{\cos^2 x}}{\cos^2 x}\,dx = -\int dx-\int\frac{\boxed{1-\sin^2 x}}{\sin^2 x}\,dx = -x-\int\left(\frac{1}{\sin^2 x}-1\right)dx =$

$= -x+\operatorname{cotg} x + x + C = \operatorname{cotg} x + C$

Př 22) $\int\frac{2x-3}{x\cdot\sqrt{x}}\,dx = \int\frac{2x}{x\sqrt{x}}\,dx-\int\frac{3}{x\cdot\sqrt{x}}\,dx = 2\int x^{-\frac{1}{2}}\,dx-3\int\frac{1}{x^{\frac{3}{2}}}\,dx =$

$= 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}}-3\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}+C = 4\sqrt{x}+6\frac{1}{\sqrt{x}}+C$

Př 23) $\int\frac{3-5\sqrt{1-x^2}}{\sqrt{1-x^2}}\,dx = 3\int\frac{dx}{\sqrt{1-x^2}}-5\int\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}\,dx = 3\arcsin x-5x+C$

Pr 24) $\int \frac{x^4-5}{1+x^2}\,dx = \int \frac{x^4-1-4}{1+x^2}\,dx = \int \frac{(x^2-1)(x^2+1)}{1+x^2}\,dx - 4\int \frac{dx}{1+x^2} =$

$= \int (x^2-1)\,dx - 4\operatorname{arctg} x = \frac{x^3}{3} - x - 4\operatorname{arctg} x + C$

25) $\int (6x^5 - 15x^4 + 8x^3 + \ln 2)\,dx = 6\int x^5 dx - 15\int x^4 dx + 8\int x^3 dx + \ln 2 \int dx =$

$= 6\frac{x^6}{6} - 15\frac{x^5}{5} + 8\frac{x^4}{4} + x\cdot \ln 2 + C = x^6 - 3x^5 + 2x^4 + x\ln 2 + C$

26) $\int \frac{(x-5)^2}{x^2}\,dx = \int \frac{x^2-10x+25}{x^2}\,dx = \int \frac{x^2}{x^2}dx - 10\int \frac{x}{x^2}dx + 25\int \frac{1}{x^2}dx =$

$= \int dx - 10\int \frac{1}{x}dx + 25\int x^{-2}dx = x - 10\ln|x| + 25\frac{x^{-1}}{-1} + C = x - 10\ln|x| - \frac{25}{x} + C$

27) $\int 3e^x\left(2 - \frac{e^{-x}}{x^4}\right)dx = \int \left(6e^x - \frac{3}{x^4}\right)dx = 6\int e^x dx - 3\int x^{-4} dx =$

$= 6e^x - 3\frac{x^{-3}}{-3} + C = 6e^x + \frac{1}{x^3} + C$

28) $\int (3\sin x + 5\cos x)\,dx = 3\int \sin x\, dx + 5\int \cos x\, dx = -3\cos x + 5\sin x + C$

29) $\int \frac{\cos 2x}{\cos^2 x}dx = \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x}dx = \int \frac{\cos^2 x}{\cos^2 x}dx - \int \frac{\sin^2 x}{\cos^2 x}dx =$

$= \int dx - \int \frac{1-\cos^2 x}{\cos^2 x}dx = x - \int \frac{1}{\cos^2 x}dx + \int \frac{\cos^2 x}{\cos^2 x}dx = x - tg x + x + C =$

$= 2x - tg x + C$

30) $\int (8x^7 + \sqrt[3]{x} - 4)dx = 8\int x^7 dx + \int x^{\frac{1}{3}}dx - 4\int dx = 8\frac{x^8}{8} + \frac{x^{\frac{4}{3}}}{\frac{4}{3}} - 4x + C =$

$= x^8 + \frac{3}{4}x^{\frac{4}{3}} - 4x + C$

31) $\int (2tg x + 3 cotg x)^2 dx = \int (4tg^2 x + 12\underbrace{tg x \cdot cotg x}_{=1} + 9cotg^2 x)dx =$

$= 4\int tg^2 x dx + 12\int dx + 9\int cotg^2 x dx = 4\int \frac{\sin^2 x}{\cos^2 x}dx + 12x + 9\int \frac{\cos^2 x}{\sin^2 x}dx =$

$= 4\int \frac{1-\cos^2 x}{\cos^2 x}dx + 12x + 9\int \frac{1-\sin^2 x}{\sin^2 x}dx = 4\int \left(\frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}\right)dx +$

$+ 12x + 9\int \left(\frac{1}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}\right)dx = 4tg x - 4x + 12x + 9(-cotg x) - 9x + C =$

$= 4tg x - x - 9cotg x + C$

32) $\int \frac{x^3 - 6x + 1}{x}dx = \int x^2 dx - 6\int dx + \int \frac{1}{x}dx = \frac{x^3}{3} - 6x + \ln|x| + C$

33) $\int \frac{3x^2-2}{x^2+1}\,dx = 3\int \frac{x^2}{x^2+1}\,dx - 2\int \frac{dx}{x^2+1} = 3\int \frac{x^2+1-1}{x^2+1}\,dx - 2\,\text{arctg}\,x =$

$= 3\int \left(\frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\right) dx - 2\,\text{arctg}\,x = 3x - 3\,\text{arctg}\,x - 2\,\text{arctg}\,x + C =$

$= 3x - 5\,\text{arctg}\,x + C$

34) $\int (x^4+3)^2\,dx = \int (x^8 + 6x^4 + 9)\,dx = \frac{x^9}{9} + 6\frac{x^5}{5} + 9x + C$

35) $\int \frac{\cos 2x}{\cos^2 x \cdot \sin^2 x}\,dx = \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \cdot \sin^2 x}\,dx = \int \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x} \cdot \sin^2 x}\,dx - \int \frac{\cancel{\sin^2 x}}{\cos^2 x \cdot \cancel{\sin^2 x}}\,dx =$

$= \int \frac{dx}{\sin^2 x} - \int \frac{dx}{\cos^2 x} = -\text{cotg}\,x - \text{tg}\,x + C$

36) $\int \frac{1}{\sin^2 x \cos^2 x}\,dx = \int \frac{\cos^2 x + \sin^2 x}{\sin^2 x \cdot \cos^2 x}\,dx = \text{tg}\,x - \text{cotg}\,x + C$

37) $\int \sin^2 \frac{x}{2}\,dx = \int \frac{1-\cos 2\frac{x}{2}}{2}\,dx = \int \frac{1}{2}\,dx - \int \frac{\cos x}{2}\,dx =$

$= \frac{1}{2}x - \frac{1}{2}\sin x + C = \frac{x - \sin x}{2} + C$

38) $\int \text{tg}^2 x\,dx = \int \frac{\sin^2 x}{\cos^2 x}\,dx = \int \frac{1-\cos^2 x}{\cos^2 x}\,dx = \int \frac{1}{\cos^2 x}\,dx - \int dx = \text{tg}\,x - x + C$

39) $$\int\left(\frac{1}{x^2}+\frac{1}{\sqrt{x}}+1\right)dx=\int x^{-2}dx+\int x^{-\frac{1}{2}}dx+\int dx=\frac{x^{-1}}{-1}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+x+C=$$

$$=-\frac{1}{x}+2\sqrt{x}+x+C$$

40) $$\int\left(5x\sqrt{x}+\frac{1}{2\sqrt{x^3}}-\frac{1}{x}\right)dx=5\int x^{\frac{3}{2}}dx+\frac{1}{2}\int x^{-\frac{3}{2}}dx-\int\frac{1}{x}dx=$$

$$=5\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+\frac{1}{2}\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}-\ln|x|+C=5\sqrt{x^5}\cdot\frac{2}{5}-\frac{1}{2}\cdot\frac{1}{\sqrt{x}}\cdot 2-\ln|x|+C=$$

$$=2\sqrt{x^5}-\frac{1}{\sqrt{x}}-\ln|x|+C$$

41) $$\int(\sqrt{x}+1)\cdot(x-\sqrt{x}+1)dx=\int(x\sqrt{x}-\sqrt{x}\cdot\sqrt{x}+\sqrt{x}+x-\sqrt{x}+1)dx=$$

$$=\int(x\sqrt{x}+1)dx=\int x^{\frac{3}{2}}dx+\int dx=\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+x+C=\frac{2}{5}\sqrt{x^5}+x+C$$

42) $$\int x^2(2-x)^2dx=\int x^2(4-4x+x^2)dx=\int(4x^2-4x^3+x^4)dx=$$

$$=4\int x^2dx-4\int x^3dx+\int x^4dx=4\frac{x^3}{3}-4\frac{x^4}{4}+\frac{x^5}{5}+C=$$

$$=\frac{4}{3}x^3-x^4+\frac{1}{5}x^5+C$$

43) $$\int\left(\frac{1-x}{x}\right)^2 dx=\int\frac{(1-x)^2}{x^2}dx=\int\frac{1-2x+x^2}{x^2}dx=\int\frac{1}{x^2}dx-2\int\frac{x}{x^2}dx+\int dx=$$

$$=\frac{x^{-1}}{-1}-2\ln|x|+x+C=-\frac{1}{x}-2\ln|x|+x+C$$

44) $$\int\frac{(1+x)^2}{x\sqrt{x}}dx=\int\frac{1+2x+x^2}{x\cdot x^{\frac{1}{2}}}dx=\int x^{-\frac{3}{2}}dx+2\int\frac{x^1}{x^{\frac{3}{2}}}dx+\int\frac{x^2}{x^{\frac{3}{2}}}dx=$$

$$=\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}+2\int x^{-\frac{1}{2}}dx+\int x^{\frac{1}{2}}dx=\frac{-2}{\sqrt{x}}+2\frac{\sqrt{x}}{\frac{1}{2}}+\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+C=$$

$$=-\frac{2}{\sqrt{x}}+4\sqrt{x}+\frac{2}{3}\sqrt{x^3}+C$$

45) $$\int\frac{5+xe^x}{x}dx=5\int\frac{1}{x}dx+\int\frac{xe^x}{x}dx=5\ln|x|+e^x+C$$

46) $$\int e^x\cdot\left(2+\frac{e^{-x}}{x^4}\right)dx=\int\left(2e^x+\frac{\overset{=1}{e^{-x}e^x}}{x^4}\right)dx=2\int e^x dx+\int\frac{1}{x^4}dx=$$

$$=2e^x+\frac{x^{-3}}{-3}+C=2e^x-\frac{1}{3x^3}+C$$

47) $$\int\frac{4^x}{5^x}dx=\int\left(\frac{4}{5}\right)^x dx=\frac{\left(\frac{4}{5}\right)^x}{\ln\frac{4}{5}}+C=\frac{1}{\ln 4-\ln 5}\left(\frac{4}{5}\right)^x+C$$

48) $\int \frac{e^{2x}+5e^{x}}{6e^{x}}dx = \int \frac{\cancel{e^{x}}(e^{x}+5)}{6\cancel{e^{x}}}dx = \frac{1}{6}\int (e^{x}+5)dx =$

$= \frac{1}{6}e^{x} + \frac{5}{6}x + C$

49) $\int (2\cdot 3^{x} - 3\cdot 2^{x})dx = 2\int 3^{x}dx - 3\int 2^{x}dx = 2\frac{3^{x}}{\ln 3} - 3\frac{2^{x}}{\ln 2} + C$

50) $\int 9^{x}\cdot e^{x}dx = \int (9e)^{x}dx = \frac{(9e)^{x}}{\ln 9e} + C = \frac{(9e)^{x}}{\ln 9 + \ln e} + C = \frac{(9e)^{x}}{\ln 9 + 1} + C$

51) $\int \frac{2^{x}-3^{x}}{6^{x}}dx = \int \left(\frac{2}{6}\right)^{x}dx - \int \left(\frac{3}{6}\right)^{x}dx = \int \left(\frac{1}{3}\right)^{x}dx - \int \left(\frac{1}{2}\right)^{x}dx =$

$= \frac{\left(\frac{1}{3}\right)^{x}}{\ln\frac{1}{3}} - \frac{\left(\frac{1}{2}\right)^{x}}{\ln\frac{1}{2}} + C = \left(\frac{1}{3}\right)^{x}\cdot\frac{1}{\ln 1 - \ln 3} - \left(\frac{1}{2}\right)^{x}\cdot\frac{1}{\ln 1 - \ln 2} + C$

52) $\int \left(\frac{1}{\pi x} + a^{\pi x}\right)dx = \frac{1}{\pi}\int \frac{1}{x}dx + \int (a^{\pi})^{x}dx = \frac{1}{\pi}\ln|x| + \frac{(a^{\pi})^{x}}{\ln(a^{\pi})} + C =$

$= \frac{\ln|x|}{\pi} + \frac{a^{\pi x}}{\pi \ln a} + C$

53) $$\int \frac{(1+\sqrt{x})^2}{6x}\,dx = \int \frac{1+2\sqrt{x}+x}{6x}\,dx = \frac{1}{6}\int \frac{1}{x}\,dx + \frac{1}{3}\int \frac{\sqrt{x}}{x}\,dx + \frac{1}{6}\int dx =$$

$$= \frac{1}{6}\ln|x| + \frac{1}{3}\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x}{6} + C = \frac{\ln|x|}{6} + \frac{2}{3}\sqrt{x} + \frac{x}{6} + C$$

54) $$\int \frac{(x^m-x^n)^2}{\sqrt{x}}\,dx = \int \frac{x^{2m}-2x^{m+n}+x^{2n}}{x^{\frac{1}{2}}}\,dx = \int \frac{x^{2m}}{x^{\frac{1}{2}}}\,dx - 2\int \frac{x^{m+n}}{x^{\frac{1}{2}}}\,dx + \int \frac{x^{2n}}{x^{\frac{1}{2}}}\,dx$$

$$= \int x^{2m-\frac{1}{2}}\,dx - 2\int x^{m+n-\frac{1}{2}}\,dx + \int x^{2n-\frac{1}{2}}\,dx =$$

$$= \int x^{\frac{4m-1}{2}}\,dx - 2\int x^{\frac{2m+2n-1}{2}}\,dx + \int x^{\frac{4n-1}{2}}\,dx = \frac{x^{\frac{4m-1}{2}+1}}{\frac{4m-1}{2}+1} - 2\frac{x^{\frac{2m+2n-1}{2}+1}}{\frac{2m+2n-1}{2}+1} +$$

$$+ \frac{x^{\frac{4n-1}{2}+1}}{\frac{4n-1}{2}+1} + C = \frac{x^{2m+\frac{1}{2}}}{2m+\frac{1}{2}} - 2\frac{x^{m+n+\frac{1}{2}}}{m+n+\frac{1}{2}} + \frac{x^{2n+\frac{1}{2}}}{2n+\frac{1}{2}} + C$$

55) $$\int \frac{\sqrt{2}}{\sin^2 x\cdot\cos^2 x}\,dx = \sqrt{2}\int \frac{1}{\sin^2 x\cdot\cos^2 x}\,dx = \sqrt{2}\int \frac{\sin^2 x+\cos^2 x}{\sin^2 x\cdot\cos^2 x}\,dx$$

$$= \sqrt{2}\int\left(\frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}\cdot\cos^2 x} + \frac{\cancel{\cos^2 x}}{\sin^2 x\cdot\cancel{\cos^2 x}}\right)dx = \sqrt{2}\int \frac{1}{\cos^2 x}\,dx + \sqrt{2}\int \frac{1}{\sin^2 x}\,dx =$$

$$= \sqrt{2}\,(\mathrm{tg}\,x - \mathrm{cotg}\,x) + C$$

56) $2\int x^3 dx - \frac{1}{2}\int x\, dx + \int \frac{1}{x} dx = 2\frac{x^4}{4} - \frac{1}{2}\cdot\frac{x^2}{2} + \ln|x| + c =$

$= \frac{1}{2}x^4 - \frac{1}{4}x^2 + \ln|x| + c$

57) $\int\left(\frac{1}{x^{\frac{3}{4}}} + \frac{1}{x^{\frac{1}{2}}}\right)dx = \int x^{-\frac{3}{4}}dx + \int x^{-\frac{1}{2}}dx = \frac{x^{\frac{1}{4}}}{\frac{1}{4}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = 4\sqrt[4]{x} + 2\sqrt{x} + c$

58) $\int \frac{(1+\sqrt{x})^2}{6x}dx = \int \frac{1+2\sqrt{x}+x}{6x}dx = \int\frac{dx}{6x} + 2\int\frac{\sqrt{x}}{6x}dx + \int\frac{x}{6x}dx =$

$= \frac{1}{6}\int\frac{dx}{x} + \frac{1}{3}\int\frac{1}{\sqrt{x}}dx + \frac{1}{6}\int dx = \frac{1}{6}\ln|x| + \frac{1}{3}\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{1}{6}x + c =$

$= \frac{\ln|x|}{6} + \frac{1}{3}\cdot\sqrt{x}\cdot 2 + \frac{x}{6} + c = \frac{\ln|x|}{6} + \frac{2\sqrt{x}}{3} + \frac{x}{6} + c$

59) $\int 10^x\cdot e^x dx = \int (10e)^x dx = \frac{(10e)^x}{\ln 10e} + c = \frac{(10e)^x}{\ln 10 + \ln e} + c =$

$= \frac{(10e)^x}{\ln 10 + 1} + c$

60) $\int\frac{(2^x-3^x)^2}{6x} = \int\frac{2^{2x} - 2\cdot(3^x\cdot 2^x) + 3^{2x}}{6^x}dx = \int\frac{2^{2x} - 2\cdot 6^x + 3^{2x}}{6^x}dx =$

$(ab)^m = a^m\cdot b^m$

$= \int\frac{2^{2x}}{6^x}dx - \int\frac{2\cdot 6^x}{6^x}dx + \int\frac{3^{2x}}{6^x} =$

$= \int\left(\frac{2^2}{6}\right)^x dx - \int 2\,dx + \int\left(\frac{3^2}{6}\right)^x dx = \int\left(\frac{2}{3}\right)^x dx - 2\int dx + \int\left(\frac{3}{2}\right)^x dx =$

$= \frac{\left(\frac{2}{3}\right)^x}{\ln\frac{2}{3}} - 2x + \frac{\left(\frac{3}{2}\right)^x}{\ln\frac{3}{2}} + c$

61) $\int \operatorname{cotg}^2 x\,dx = \int \frac{\cos^2 x}{\sin^2 x}\,dx = \int \frac{1-\sin^2 x}{\sin^2 x}\,dx = \int \frac{1}{\sin^2 x}\,dx - \int dx =$

$= -\operatorname{cotg} x - x + C$

62) $\int \frac{5\operatorname{tg}^2 x - 1}{\sin^2 x}\,dx = \int \frac{5\cdot\frac{\sin^2 x}{\cos^2 x} - 1}{\underset{1}{\underline{\sin^2 x}}}\,dx = 5\int \frac{\cancel{\sin^2 x}}{\cos^2 x}\cdot\frac{1}{\cancel{\sin^2 x}}\,dx - \int \frac{dx}{\sin^2 x} =$

$= 5\int \frac{dx}{\cos^2 x} - \int \frac{dx}{\sin^2 x} = 5\operatorname{tg} x + \operatorname{cotg} x + C$

63) $\int \left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^2 dx = \int \left(\underbrace{\sin^2\frac{x}{2} + \cos^2\frac{x}{2}}_{=1} - 2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}\right) dx =$

$= \int \left(1 - 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}\right)dx = \int \left(1 - \sin\left(2\left(\frac{x}{2}\right)\right)\right)dx = \int (1-\sin x)\,dx =$

$\boxed{\sin 2\alpha = 2\cdot\sin\alpha\cdot\cos\alpha}$ $= \int dx - \int \sin x\,dx = x + \cos x + C$