Neurčitý integral

$$\int x \cdot \operatorname{arctg}\sqrt{2x^2-1}\,dx$$

$$= \left| \begin{array}{l} \sqrt{2x^2-1} = t \\ 2x^2-1 = t^2 \\ 4x\,dx = 2t\,dt \\ x\,dx = \frac{t}{2}\,dt \end{array} \right| = \int \frac{t}{2} \cdot \operatorname{arctg} t\,dt = \left| \begin{array}{ll} u' = \frac{1}{2}t & v = \operatorname{arctg} t \\ u = \frac{1}{2} \cdot \frac{t^2}{2} & v' = \frac{1}{1+t^2} \end{array} \right|$$

$$= \frac{t^2}{4} \cdot \operatorname{arctg} t - \int \frac{t^2}{4} \cdot \frac{1}{1+t^2}\,dt = \frac{t^2}{4}\operatorname{arctg} t - \frac{1}{4}\int \frac{t^2}{1+t^2}\,dt =$$

$$= \frac{t^2}{4}\operatorname{arctg} t - \frac{1}{4}\int \frac{t^2+1-1}{1+t^2}\,dt =$$

$$= \frac{t^2}{4} \cdot \operatorname{arctg} t - \frac{1}{4}\int \frac{1+t^2}{1+t^2}\,dt + \frac{1}{4}\int \frac{1}{1+t^2}\,dt =$$

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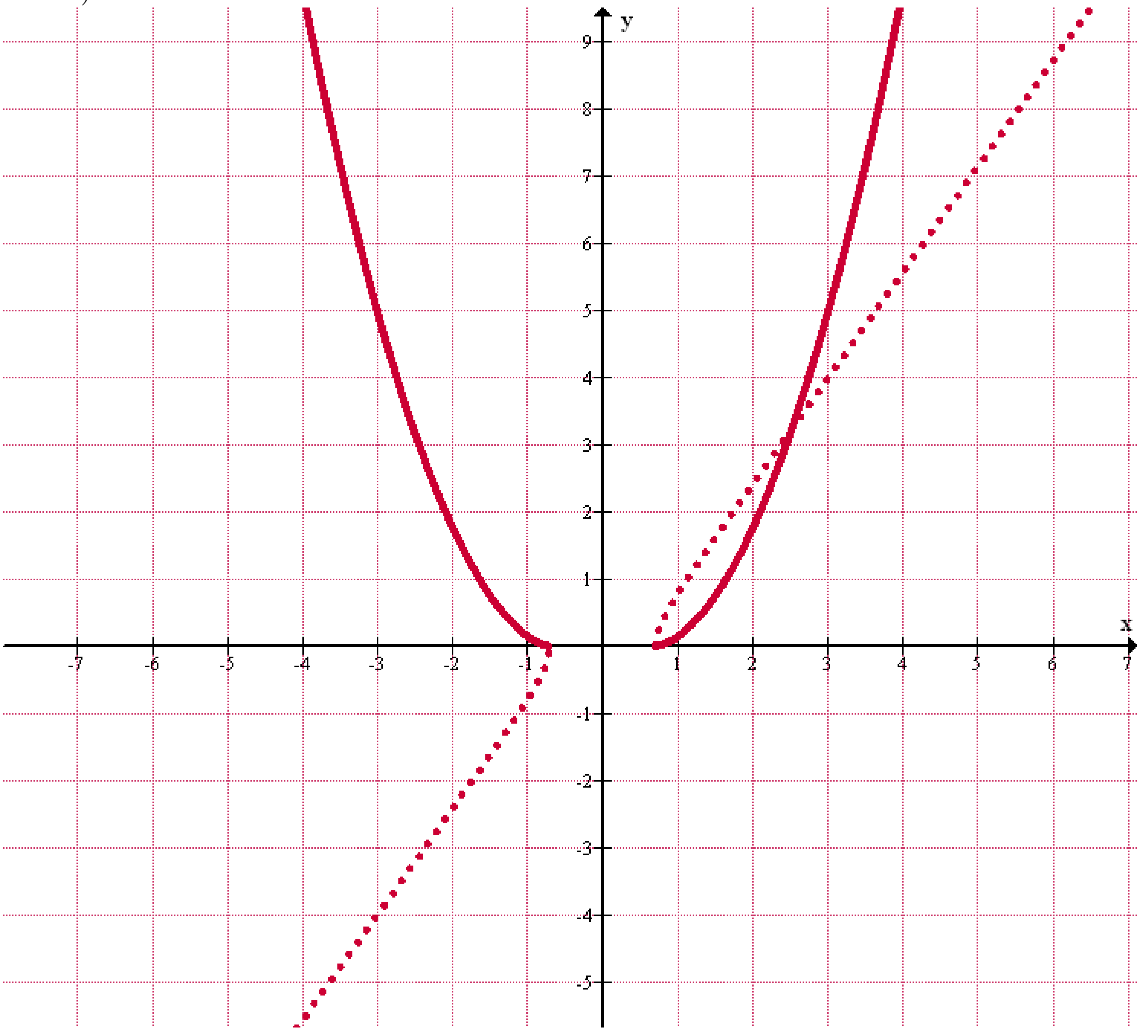
$$= \frac{t^2}{4} \cdot \operatorname{arctg} t - \frac{1}{4}t + \frac{1}{4} \cdot \operatorname{arctg} t + C$$

Substituce zpět:

$$\frac{2x^2-1}{4} \cdot \operatorname{arctg}\sqrt{2x^2-1} - \frac{\sqrt{2x^2-1}}{4} + \frac{1}{4}\operatorname{arctg}\sqrt{2x^2-1} + C =$$

$$= \frac{x^2}{2}\operatorname{arctg}\sqrt{2x^2-1} - \frac{\sqrt{2x^2-1}}{4} + C$$

Obrázek 1. Grafické znázornění funkce (tečkovaná) a jejího integrálu (plná čára) s nulovou konstantou



Zdroj: program Graph