1) $\int x\cdot \cos dx = \left| \begin{array}{ll} u' = \cos x & v = x \\ u = \sin x & v' = 1 \end{array} \right| = x\cdot \sin x - \int 1\cdot \sin x\, dx =$

$= x\cdot \sin x + \cos x + C$

2) $\int x^3 \cdot \ln x\, dx = \left| \begin{array}{ll} u' = x^3 & v = \ln x \\ u = \frac{x^4}{4} & v' = \frac{1}{x} \end{array} \right| = \frac{x^4}{4}\ln x - \int \frac{x^4}{4}\cdot\frac{1}{x}\, dx =$

$= \frac{x^4}{4}\ln x - \frac{1}{4}\int x^3 dx = \frac{x^4}{4}\ln x - \frac{1}{4}\cdot\frac{x^4}{4} + C = \frac{x^4}{4}\left(\ln x - \frac{1}{4}\right) + C$

3) $\int \ln x \cdot 1\, dx = \left| \begin{array}{ll} u' = 1 & v = \ln x \\ u = x & v' = \frac{1}{x} \end{array} \right| = x\ln x - \int x\cdot\frac{1}{x}\, dx = x\cdot \ln x - x + C$

4) $\int x^2 \cdot e^x\, dx = \left| \begin{array}{ll} u' = e^x & v = x^2 \\ u = e^x & v' = 2x \end{array} \right| = x^2 e^x - \int 2x\cdot e^x\, dx =$

$= x^2\cdot e^x - \left| \begin{array}{ll} u' = e^x & v = 2x \\ u = e^x & v' = 2 \end{array} \right| = x^2 e^x - \left[2xe^x - \int 2e^x dx\right] =$

$= x^2 e^x - 2xe^x + 2e^x + C$

5) $\int \sin x \cdot e^x dx = \left| \begin{array}{ll} u' = e^x & v = \sin x \\ u = e^x & v' = \cos x \end{array} \right| = e^x \cdot \sin x - \int e^x \cdot \cos x \, dx =$

$= e^x \cdot \sin x - \left| \begin{array}{ll} u' = e^x & v = \cos x \\ u = e^x & v' = -\sin x \end{array} \right| = e^x \cdot \sin x - \left[e^x \cdot \cos x + \int \sin x \cdot e^x dx \right] =$

Tento typ integrálu "vede na rovnici"!

$$\underbrace{\int \sin x \cdot e^x dx}_{\text{zadání}} = \underbrace{e^x \cdot \sin x - e^x \cdot \cos x - \int \sin x \cdot e^x dx}_{\text{dosavadní výsledek}} \qquad / + \int \sin x \cdot e^x dx$$

$$2 \int \sin x \cdot e^x dx = e^x \cdot \sin x - e^x \cdot \cos x \qquad /:2$$

$$\int \sin x \cdot e^x dx = \frac{e^x \sin x - e^x \cdot \cos x}{2} + c$$

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6) $\int x^7 \cdot \ln x \, dx = \left| \begin{array}{ll} u' = x^7 & v = \ln x \\ u = \frac{x^8}{8} & v' = \frac{1}{x} \end{array} \right| = \frac{x^8}{8} \ln x - \int \frac{x^8}{8} \cdot \frac{1}{x} dx =$

$= \frac{x^8}{8} \ln x - \frac{1}{8} \int x^7 dx = \frac{x^8}{8} \ln x - \frac{1}{8} \cdot \frac{x^8}{8} + C = \frac{x^8}{8} \cdot \left(\ln x - \frac{1}{8} \right) + c$

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7) $\int x \cdot 2^x dx = \left| \begin{array}{ll} u' = 2^x & v = x \\ u = \frac{2^x}{\ln 2} & v' = 1 \end{array} \right| = x \cdot \frac{2^x}{\ln 2} - \int \frac{2^x}{\ln 2} dx = \frac{x 2^x}{\ln 2} - \frac{1}{\ln 2} \int 2^x dx =$

$= \frac{x \cdot 2^x}{\ln 2} - \frac{1}{\ln 2} \cdot \frac{2^x}{\ln 2} + C = \frac{2^x}{\ln 2} \cdot \left(x - \frac{1}{\ln 2} \right) + c$

8) $\int (x^3+2x^2-5x)\cdot e^x dx = \left| \begin{matrix} u' = e^x & v = x^3+2x^2-5x \\ u = e^x & v' = 3x^2+4x-5 \end{matrix} \right| =$

$= (x^3+2x^2-5x)e^x - \int (3x^2+4x-5)e^x dx = (x^3+2x^2-5x)e^x - \left| \begin{matrix} u' = e^x & v = 3x^2+4x-5 \\ u = e^x & v' = 6x+4 \end{matrix} \right| =$

$= (x^3+2x^2-5x)e^x - \left[(3x^2+4x-5)e^x - \int (6x+4)e^x dx \right] =$

$= (x^3+2x^2-5x)e^x - (3x^2+4x-5)e^x + \left| \begin{matrix} u' = e^x & v = 6x+4 \\ u = e^x & v' = 6 \end{matrix} \right| =$

$= (x^3+2x^2-5x)e^x - (3x^2+4x-5)e^x + \left[(6x+4)e^x - 6\int e^x dx \right] =$

$= (x^3+2x^2-5x)e^x - (3x^2+4x-5)e^x + (6x+4)e^x - 6e^x + C =$

$= \underbrace{e^x}_{\text{vytčeno}} (x^3 + 2x^2 - 5x - 3x^2 - 4x + \boxed{5} + 6x \boxed{+4} \boxed{-6}) + C =$

$= e^x (x^3 - x^2 - 3x + 3) + C$

9) $\int (x+3)\cos x\, dx = \left| \begin{matrix} u' = \cos x & v = x+3 \\ u = \sin x & v' = 1 \end{matrix} \right| = (x+3)\sin x - \int \sin x\, dx =$

$= \sin x \cdot (x+3) + \cos x + C$

10) $\int x \cdot \operatorname{arctg} x\, dx = \left| \begin{matrix} u' = x & v = \operatorname{arctg} x \\ u = \frac{x^2+1}{2}{}^{*} & v' = \frac{1}{1+x^2} \end{matrix} \right| = \frac{x^2+1}{2}\operatorname{arctg} x - \int \frac{\cancel{x^2+1}}{2} \cdot \frac{1}{\cancel{1+x^2}} dx =$

$= \frac{x^2+1}{2}\operatorname{arctg} x - \frac{1}{2}x + C$

* Po integraci se vždy připisuje +C. V pomocné tabulce u metody per partes to nikdy neděláme, nicméně lze přičíst konstantu. A to i jako konkrétní vybranou hodnotu.

11) $\int (5-5x)\cos x \cdot dx = \left| \begin{matrix} u' = \cos x & v = 5-5x \\ u = \sin x & v' = -5 \end{matrix} \right| = (5-5x)\cdot \sin x + 5\int \sin x\, dx =$

$= (5-5x)\sin x - 5\cos x + C$

12) $\int 3^x \cdot \cos x\, dx = \left| \begin{matrix} u' = \cos x & v = 3^x \\ u = \sin x & v' = 3^x \cdot \ln 3 \end{matrix} \right| = 3^x \cdot \sin x - \ln 3 \int 3^x \cdot \sin x\, dx =$

$= 3^x \cdot \sin x - \ln 3 \left| \begin{matrix} u' = \sin x & v = 3^x \\ u = -\cos x & v' = 3^x \ln 3 \end{matrix} \right| = 3^x \cdot \sin x - \ln 3 \left[-3^x \cdot \cos x + \ln 3 \int 3^x \cdot \cos x\, dx \right] =$

Tento příklad „vede na rovnici“:

$= \underbrace{3^x \cdot \sin x + 3^x \ln 3 \cos x - \ln^2 3 \int 3^x \cdot \cos x\, dx}_{\text{dosavadní výsledek}} = \underbrace{\int 3^x \cdot \cos x\, dx}_{\text{zadání}}$

$$\int 3^x \cdot \cos x\, dx = \frac{3^x \sin x + 3^x \ln 3 \cdot \cos x}{1 + \ln^2 3} + C$$

13) $\int \frac{\ln x}{\sqrt{x}}\, dx = \int \ln x \cdot \frac{1}{\sqrt{x}}\, dx = \left| \begin{matrix} u' = \frac{1}{\sqrt{x}} & v = \ln x \\ u = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} & v' = \frac{1}{x} \end{matrix} \right| =$

$= \frac{\sqrt{x}}{\frac{1}{2}} \cdot \ln x - \int \frac{\sqrt{x}}{\frac{1}{2}} \cdot \frac{1}{x}\, dx = 2\sqrt{x} \ln x - \int 2\sqrt{x} \cdot \frac{1}{x}\, dx = 2\sqrt{x} \ln x - 2\int x^{-\frac{1}{2}}\, dx =$

$= 2\sqrt{x} \cdot \ln x - 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{x} \cdot \ln x - 4\sqrt{x} + C$

14) $\int \sin^2 x\, dx = \int \sin x \cdot \sin x\, dx = \left| \begin{array}{ll} u' = \sin x & v = \sin x \\ u = -\cos x & v' = \cos x \end{array} \right| =$

$= -\cos x \cdot \sin x + \int \cos x \cdot \cos x\, dx = -\sin x \cdot \cos x + \int (1 - \sin^2 x)\, dx =$

$= -\sin x \cdot \cos x + \int dx - \int \sin^2 x\, dx$

Tento příklad "vede na rovnici"

$\underbrace{\int \sin^2 x\, dx}_{\text{zadání}} = -\sin x \cdot \cos x + x - \int \sin^2 x\, dx \qquad / + \int \sin^2 x\, dx$

$2 \int \sin^2 x\, dx = x - \sin x \cdot \cos x \qquad / : 2$

$\int \sin^2 x\, dx = \dfrac{x - \sin x \cdot \cos x}{2} + C$

Tento příklad lze řešit i substituční metodou (viz soubor Substituční metoda, př. 12)