Neurčitý integral

$$\int x^3\left(\ln x + \frac{\cos x}{2x^3\sqrt[3]{3\cdot\sin(x)-1}}\right)dx$$

$$= \underbrace{\int x^3 \ln x\,dx}_{I_1} + \underbrace{\int \frac{\cancel{x^3}\cos x}{2\cancel{x^3}\sqrt[3]{3\sin(x)-1}}\,dx}_{I_2}$$

$$I_1 = \left|\begin{array}{ll} u' = x^3 & v = \ln x \\ u = \frac{x^4}{4} & v' = \frac{1}{x}\end{array}\right| = \frac{x^4}{4}\ln x - \int \frac{x^3}{4}\,dx = \frac{x^4}{4}\ln x - \frac{1}{4}\int x^3\,dx =$$

$$= \frac{x^4}{4}\ln x - \frac{1}{4}\cdot\frac{x^4}{4} + C = \frac{x^4}{4}\left(\ln x - \frac{1}{4}\right) + C$$

$$I_2 = \frac{1}{2}\int \frac{\cos x}{\sqrt[3]{3\sin(x)-1}}\,dx = \frac{1}{2}\left|\begin{array}{l} \sqrt[3]{3\sin x - 1} = t \quad /^3 \\ 3\sin x - 1 = t^3 \\ 3\cos x\,dx = 3t^2\,dt \\ \cos x\,dx = t^2\,dt \end{array}\right| = \frac{1}{2}\int \frac{t^2}{t}\,dt =$$

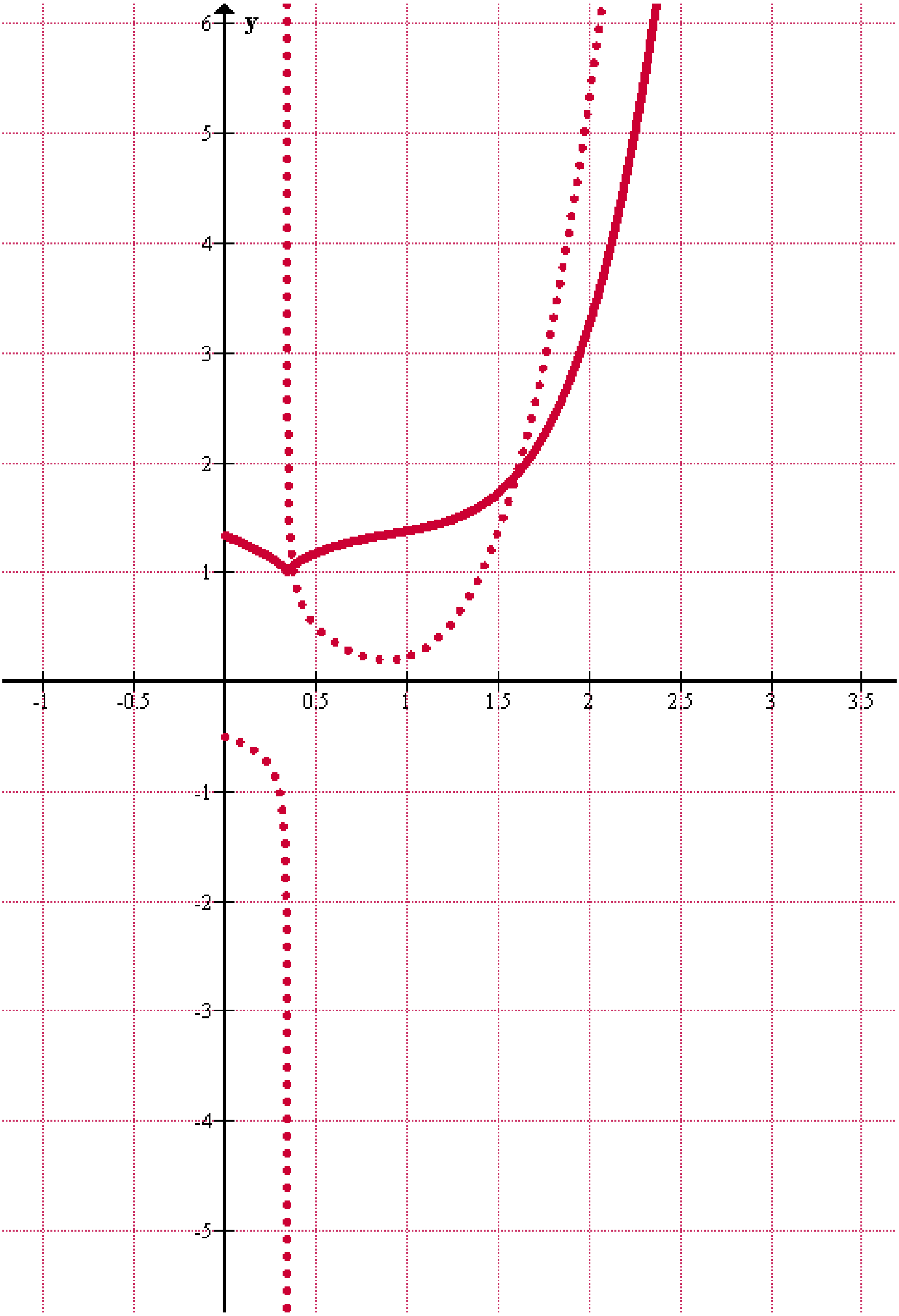
$$= \frac{1}{2}\int t\,dt = \frac{1}{2}\cdot\frac{t^2}{2} + C = \frac{t^2}{4} + C.$$ Substituce zpět:

$$\frac{1}{4}\left(\sqrt[3]{3\sin x - 1}\right)^2 + C$$

$$I = I_1 + I_2$$

$$\frac{x^4}{4}\left(\ln x - \frac{1}{4}\right) + \frac{1}{4}\sqrt[3]{(3\sin x - 1)^2} + C$$

Obrázek 1. Grafické znázornění funkce (tečkovaná) a jejího integrálu (plná čára) s nulovou konstantou



Zdroj: program Graph