Neurčitý integrál

$$\int x(\ln x + x)\,dx$$

$$= \int (x\ln x + x^2)\,dx = \underbrace{\int x\cdot\ln x\,dx}_{I_1} + \underbrace{\int x^2\,dx}_{I_2} =$$

I_2: $\int x^2\,dx = \underline{\frac{x^3}{3} + c}$

I_1: $\int x\cdot\ln x\,dx$ = per partes: $\left| \begin{array}{ll} u' = x & v = \ln x \\ u = \frac{x^2}{2} & v' = \frac{1}{x} \end{array} \right| =$

$$= \frac{x^2}{2}\cdot\ln x - \int \frac{x^{\cancel{2}}}{2}\cdot\frac{1}{\cancel{x}}\,dx = \frac{x^2}{2}\cdot\ln x - \frac{1}{2}\int x\,dx =$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\cdot\frac{x^2}{2} + c = \underline{\underline{\frac{x^2}{2}\left(\ln x - \frac{1}{2}\right) + c}}$$

Celý výsledek: $I_1 + I_2$

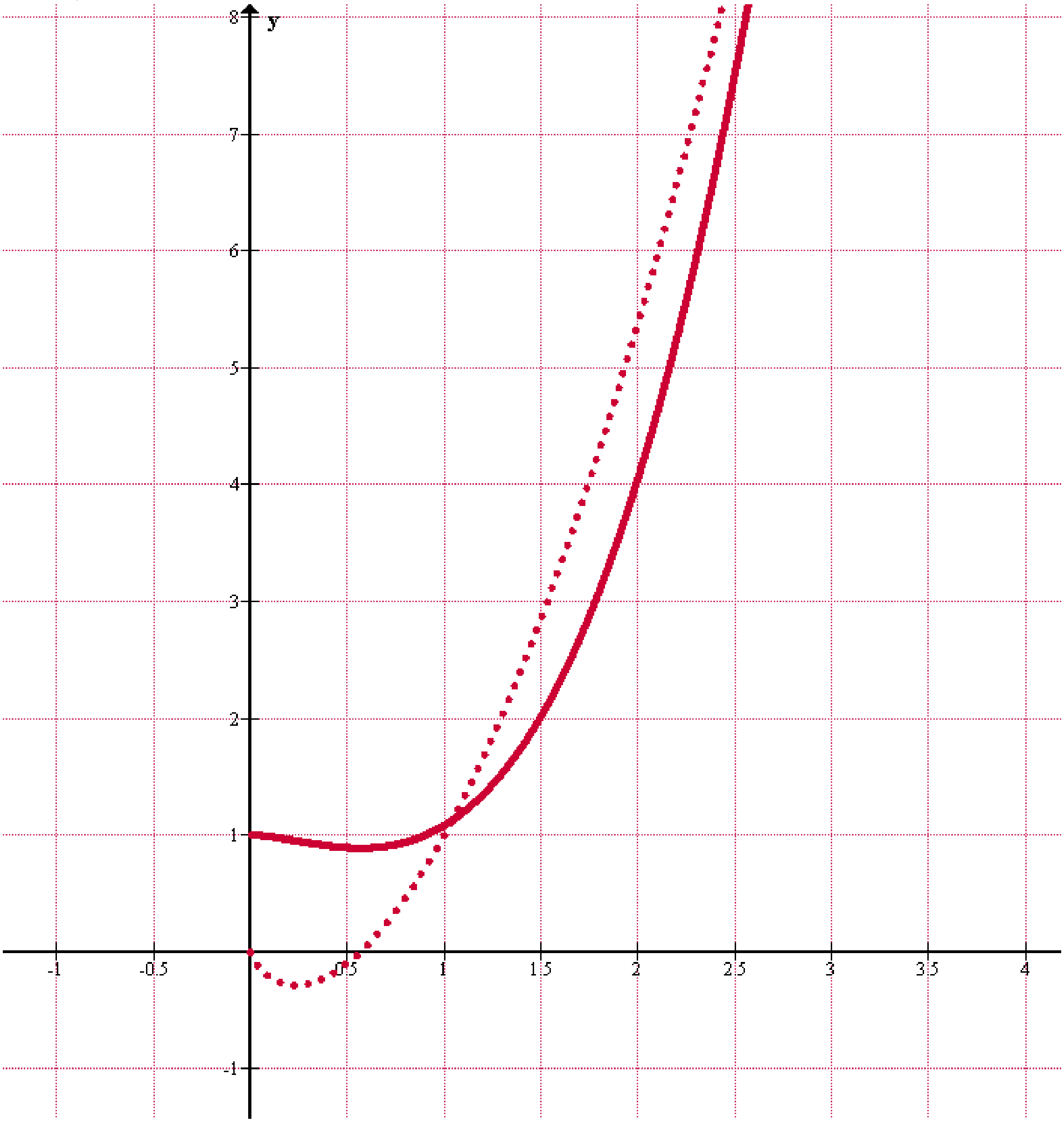
$$\underline{\underline{\frac{x^2}{2}\left(\ln x - \frac{1}{2}\right) + \frac{x^3}{3} + c}}$$

Kontrola - zpětná derivace výsledku:

$$\left(\frac{x^2}{2}\ln x - \frac{x^2}{4} + \frac{x^3}{3}\right)' = \frac{\cancel{2}x}{\cancel{2}}\ln x + \frac{x^{\cancel{2}}}{2}\cdot\frac{1}{\cancel{x}} - \frac{2x}{\cancel{4}} + \frac{\cancel{3}x^2}{\cancel{3}} =$$

$$= x\ln x + \frac{x}{\cancel{2}} - \frac{x}{\cancel{2}} + x^2 = x\cdot\ln x + x^2 = x(\ln x + x)$$

Obrázek 1. Grafické znázornění funkce (tečkovaná) a jejího integrálu (plná čára) s nulovou konstantou



Zdroj: program Graph