

SKRIPTA – DERIVACE

Příklady ze skript **Matematika I**, Slavík, V., Wolhmutová, M., 2004.

Zadání

1) $y = 7\sqrt{x} + 5\sqrt{2}$

2) $y = \frac{12\sqrt{x^3\sqrt{x}}}{5\sqrt[3]{x^4}}$

3) $y = \frac{(\sqrt{x} - 1)^2}{x}$

4) $y = \frac{x}{2x - 1}$

5) $y = x^2 \cdot 3^x$

6) $y = x \cdot \ln x - x$

7) $y = (2 - x^2) \cdot \cos x + 2x \cdot \sin x$

8) $y = (x - 1) \cdot \log_3 x$

9) $y = \frac{\operatorname{tg} x}{e^x}$

10) $y = \frac{4x + 6}{9 - 4x^2}$

11) $y = \frac{\cos x}{1 - \sin x}$

12) $y = \frac{1 + \ln x}{x}$

13) $y = \ln \left(\frac{x + 3}{x - 3} \right)$

14) $y = \operatorname{arctg} \left(\frac{1}{x} \right)$

15) $y = \operatorname{arccotg} \left(\frac{x + 1}{x - 1} \right)$

16) $y = e^{\sqrt{x+1}} + \sqrt{x+1}$

17) $y = \arcsin \sqrt{x^2 - 1}$

18) $y = \operatorname{tg}^4 x - 2 \cdot \operatorname{tg}^2 x - 4 \cdot \ln(\cos(x))$

19) $y = \ln \left(\operatorname{tg} \left(\frac{x}{2} \right) \right)$

Výsledky

1 ✓ $y' = \frac{7}{2\sqrt{x}}$

2 ✓ $y' = \frac{1}{12\sqrt{x^7}}$

3 ✓ $y' = \frac{\sqrt{x} - 1}{x^2}$

4 ✓ $y' = -\frac{1}{(2x - 1)^2}$

5 ✓ $y' = x \cdot 3^x \cdot (2 + x \cdot \ln 3)$

6 ✓ $y' = \ln x$

7 ✓ $y' = x^2 \cdot \sin x$

8 ✓ $y' = \log_3 x + \frac{1}{\ln 3} \cdot \left(1 - \frac{1}{x} \right)$

9 ✓ $y' = \frac{1 - \sin x \cdot \cos x}{e^x \cdot \cos^2 x}$

10 ✓ $y' = \frac{4}{(3 - 2x)^2}$

11 ✓ $y' = \frac{1}{1 - \sin x}$

12 ✓ $y' = -\frac{\ln x}{x^2}$

13 ✓ $y' = \frac{6}{9 - x^2}$

14 ✓ $y' = -\frac{1}{1 + x^2}$

15 ✓ $y' = \frac{1}{1 + x^2}$

16 ✓ $y' = \frac{e^{\sqrt{x+1}} + 1}{2\sqrt{x+1}}$

17 ✓ $y' = \frac{x}{\sqrt{(2 - x^2)(x^2 - 1)}}$

18 ✓ $y' = 4 \cdot \operatorname{tg}^5 x$

19 ✓ $y' = \frac{1}{\sin x}$

Zadání

20) $y = 2x - (1 - x^2) \cdot \ln \left(\frac{1+x}{1-x} \right)$

21) $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^5$

22) $y = x \cdot \operatorname{arctg} x - \frac{1}{2} \cdot \ln(1+x^2)$

23) $y = \ln \left(\operatorname{tg} \left(\frac{\pi}{4} + \frac{x}{2} \right) \right)$

24) $y = \frac{x}{8} \cdot \sqrt{16-x^2} + 2 \cdot \arcsin \frac{x}{4}$

25) $y = \sqrt{16x-x^2} + 4 \cdot \arcsin \frac{\sqrt{x}}{4}$

26) $y = \frac{1}{\sqrt{3}} \cdot \operatorname{arctg} \left(\frac{x \cdot \sqrt{3}}{1-x^2} \right)$

27) $y = x \cdot \arcsin \left(\frac{1}{x} \right) + \ln \left(x + \sqrt{x^2-1} \right)$

28) $y = \frac{1}{2} \cdot \ln \left(\operatorname{tg} \left(\frac{x}{2} \right) \right) - \frac{\cos x}{2 \sin^2 x}$

29) $y = \frac{x}{(x^2+1)^2} + \frac{3x}{2(x^2+1) + \frac{3}{2} \operatorname{arctg} x}$

30) $y = \ln \left(\frac{1-\sqrt{1-x^2}}{x} - \frac{\arcsin x}{x} \right)$

31) $y = \frac{1}{2} \cdot \ln \left(\frac{x^2}{x^2+1} \right) - \frac{\operatorname{arctg} x}{x}$

Vypočtete druhé derivace funkcí:

32) $y = x \cdot \operatorname{tg} x + \ln(\cos x)$

33) $y = \ln \left(x + \sqrt{x^2+1} \right)$

34) $y = \frac{2x \cdot \sqrt{x}}{3} \cdot \left(\ln x - \frac{2}{3} \right)$

35) $y = \ln \left(\frac{x+3}{\sqrt{x^2+4}} \right)$

36) $y = \operatorname{arctg} \left(\frac{1-x^3}{1+x^3} \right)$

37) $y = \ln \sqrt{\frac{1-x}{1+x}} + \operatorname{arctg} \left(\frac{1-x}{1+x} \right)$

38) $y = \frac{3+e^{2x}}{4-e^{2x}}$

39) $y = \ln \sqrt{\frac{1+e^{2x}}{1-e^{2x}}}$

Výsledky

20 ✓ $y' = 2x \cdot \ln \left(\frac{1+x}{1-x} \right)$

21 ✓ $y' = \frac{5(x+1)^4 \cdot (x-1)}{2x^3 \cdot \sqrt{x}}$

22 ✓ $y' = \operatorname{arctg} x$

23 ✓ $y' = \frac{1}{\cos x}$

24 ✓ $y' = \frac{\sqrt{16-x^2}}{4}$

25 ✓ $y' = \frac{10-x}{\sqrt{16x-x^2}}$

26 ✓ $y' = \frac{x^2+1}{x^4+x^2+1}$

27 ✓ $y' = \arcsin \left(\frac{1}{x} \right)$

28 ✓ $y' = \frac{1}{\sin^3 x}$

29 ✓ $y' = \frac{4}{(x^2+1)^3}$

30 ✓ $y' = \frac{\arcsin x}{x^2}$

31 ✓ $y' = \frac{\operatorname{arctg} x}{x^2}$

32 ✓ $y' = \frac{2x \cdot \operatorname{tg} x + 1}{x^2}$

33 ✓ $y' = \frac{-x}{\sqrt{(x^2+1)^3}}$

34 ✓ $y' = \frac{\ln x + 2}{2 \cdot \sqrt{x}}$

35 ✓ $y' = \frac{6x^3 - 3x^2 - 24x - 52}{((x+3) \cdot (x^2+4))^2}$

36 ✓ $y' = \frac{6x \cdot (2x^6 - 1)}{(1+x^6)^2}$

37 ✓ $y' = \frac{-8x^3}{(x^4-1)^2}$

38 ✓ $y' = \frac{28e^{2x} \cdot (4+e^{2x})}{(4-e^{2x})^3}$

39 ✓ $y' = \frac{4e^{2x} \cdot (1+e^{4x})}{(1-e^{4x})^2}$

Zadání

Vypočtete $f'(4)$ pro funkci

$$40) \quad f(x) = \frac{\sqrt{x}}{1 + 2 \cdot \sqrt{x}}$$

Vypočtete $f'(0)$ a $f''(0)$ pro funkci

$$41) \quad f(x) = \frac{x}{2} \cdot \sqrt{9 - x^2} \cdot \left(\frac{9}{2}\right) \cdot \arcsin \frac{x}{3}$$

Vypočtete $f'(5)$ a $f''(5)$ pro funkci

$$42) \quad f(x) = x^2 - x \cdot \sqrt{x^2 - 9} + \ln(x + \sqrt{x^2 - 9})$$

Ve kterých intervalech je derivace zadané funkce kladná?

$$43) \quad y = \operatorname{tg}^2 x + 2 \cdot \ln(\operatorname{tg} x)$$

Pro které x je derivace zadané funkce rovna nule?

$$44) \quad y = (4x - 1) \cdot e^{\frac{1}{x}}$$

Pro která x je derivace zadané funkce rovna nule?

$$45) \quad y = \sin^3 x - 3 \cdot \sin^2 x + 3 \cdot \sin x$$

Pro která x platí $f'(x) = 4$ u zadané funkce?

$$46) \quad y = \frac{4 \cdot \sin x}{1 + \cos x}$$

Výsledky

$$40) \quad \checkmark \quad f'(4) = 0,01$$

$$41) \quad \checkmark \quad f'(0) = 3, \quad f''(0) = 0$$

$$42) \quad \checkmark \quad f'(5) = 0, \quad f''(5) = \frac{1}{8}$$

$$43) \quad \checkmark \quad x \in \left(2k \cdot \frac{\pi}{2}, (2k + 1) \cdot \frac{\pi}{2}\right)$$

$$44) \quad \checkmark \quad x = \frac{1}{2}$$

$$45) \quad \checkmark \quad x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$$

$$46) \quad \checkmark \quad x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$$