

SKRIPTA – DERIVACE

Příklady ze skript **Matematika I**, Slavík, V., Wolhmutová, M., 2004.

Zadání

$$1) \quad y = 7\sqrt{x} + 5\sqrt{2}$$

$$2) \quad y = \frac{12\sqrt{x^3\sqrt{x}}}{5\sqrt[3]{x^4}}$$

$$3) \quad y = \frac{(\sqrt{x} - 1)^2}{x}$$

$$4) \quad y = \frac{x}{2x - 1}$$

$$5) \quad y = x^2 \cdot 3^x$$

$$6) \quad y = x \cdot \ln x - x$$

$$7) \quad y = (2 - x^2) \cdot \cos x + 2x \cdot \sin x$$

$$8) \quad y = (x - 1) \cdot \log_3 x$$

$$9) \quad y = \frac{\operatorname{tg} x}{e^x}$$

$$10) \quad y = \frac{4x + 6}{9 - 4x^2}$$

$$11) \quad y = \frac{\cos x}{1 - \sin x}$$

$$12) \quad y = \frac{1 + \ln x}{x}$$

$$13) \quad y = \ln \left(\frac{x + 3}{x - 3} \right)$$

$$14) \quad y = \operatorname{arctg} \left(\frac{1}{x} \right)$$

$$15) \quad y = \operatorname{arccotg} \left(\frac{x + 1}{x - 1} \right)$$

$$16) \quad y = e^{\sqrt{x+1}} + \sqrt{x+1}$$

$$17) \quad y = \arcsin \sqrt{x^2 - 1}$$

$$18) \quad y = \operatorname{tg}^4 x - 2 \cdot \operatorname{tg}^2 x - 4 \cdot \ln(\cos(x))$$

$$19) \quad y = \ln \left(\operatorname{tg} \left(\frac{x}{2} \right) \right)$$

Výsledky

$$1 \checkmark \quad y' = \frac{7}{2\sqrt{x}}$$

$$2 \checkmark \quad y' = \frac{1}{\sqrt[12]{x^7}}$$

$$3 \checkmark \quad y' = \frac{\sqrt{x} - 1}{x^2}$$

$$4 \checkmark \quad y' = -\frac{1}{(2x - 1)^2}$$

$$5 \checkmark \quad y' = x \cdot 3^x \cdot (2 + x \cdot \ln 3)$$

$$6 \checkmark \quad y' = \ln x$$

$$7 \checkmark \quad y' = x^2 \cdot \sin x$$

$$8 \checkmark \quad y' = \log_3 x + \frac{1}{\ln 3} \cdot \left(1 - \frac{1}{x} \right)$$

$$9 \checkmark \quad y' = \frac{1 - \sin x \cdot \cos x}{e^x \cdot \cos^2 x}$$

$$10 \checkmark \quad y' = \frac{4}{(3 - 2x)^2}$$

$$11 \checkmark \quad y' = \frac{1}{1 - \sin x}$$

$$12 \checkmark \quad y' = -\frac{\ln x}{x^2}$$

$$13 \checkmark \quad y' = \frac{6}{9 - x^2}$$

$$14 \checkmark \quad y' = -\frac{1}{1 + x^2}$$

$$15 \checkmark \quad y' = \frac{1}{1 + x^2}$$

$$16 \checkmark \quad y' = \frac{e^{\sqrt{x+1}} + 1}{2\sqrt{x+1}}$$

$$17 \checkmark \quad y' = \frac{x}{\sqrt{(2 - x^2)(x^2 - 1)}}$$

$$18 \checkmark \quad y' = 4 \cdot \operatorname{tg}^5 x$$

$$19 \checkmark \quad y' = \frac{1}{\sin x}$$

Zadání

$$20) \quad y = 2x - (1 - x^2) \cdot \ln \left(\frac{1+x}{1-x} \right)$$

$$21) \quad y = \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^5$$

$$22) \quad y = x \cdot \operatorname{arctg} x - \frac{1}{2} \cdot \ln(1 + x^2)$$

$$23) \quad y = \ln \left(\operatorname{tg} \left(\frac{\pi}{4} + \frac{x}{2} \right) \right)$$

$$24) \quad y = \frac{x}{8} \cdot \sqrt{16 - x^2} + 2 \cdot \arcsin \frac{x}{4}$$

$$25) \quad y = \sqrt{16x - x^2} + 4 \cdot \arcsin \frac{\sqrt{x}}{4}$$

$$26) \quad y = \frac{1}{\sqrt{3}} \cdot \operatorname{arctg} \left(\frac{x \cdot \sqrt{3}}{1 - x^2} \right)$$

$$27) \quad y = x \cdot \arcsin \left(\frac{1}{x} \right) + \ln \left(x + \sqrt{x^2 - 1} \right)$$

$$28) \quad y = \frac{1}{2} \cdot \ln \left(\operatorname{tg} \left(\frac{x}{2} \right) \right) - \frac{\cos x}{2 \sin^2 x}$$

$$29) \quad y = \frac{x}{(x^2 + 1)^2} + \frac{3x}{2(x^2 + 1) + \frac{3}{2} \operatorname{arctg} x}$$

$$30) \quad y = \ln \left(\frac{1 - \sqrt{1 - x^2}}{x} - \frac{\arcsin x}{x} \right)$$

$$31) \quad y = \frac{1}{2} \cdot \ln \left(\frac{x^2}{x^2 + 1} \right) - \frac{\operatorname{arctg} x}{x}$$

Vypočtete druhé derivace funkcí:

$$32) \quad y = x \cdot \operatorname{tg} + \ln(\cos x)$$

$$33) \quad y = \ln \left(x + \sqrt{x^2 + 1} \right)$$

$$34) \quad y = \frac{2x \cdot \sqrt{x}}{3} \cdot \left(\ln x - \frac{2}{3} \right)$$

$$35) \quad y = \ln \left(\frac{x+3}{\sqrt{x^2+4}} \right)$$

$$36) \quad y = \operatorname{arctg} \left(\frac{1-x^3}{1+x^3} \right)$$

$$37) \quad y = \ln \sqrt{\frac{1-x}{1+x}} + \operatorname{arctg} \left(\frac{1-x}{1+x} \right)$$

$$38) \quad y = \frac{3 + e^{2x}}{4 - e^{2x}}$$

$$39) \quad y = \ln \sqrt{\frac{1 + e^{2x}}{1 - e^{2x}}}$$

Výsledky

$$20 \quad \checkmark \quad y' = 2x \cdot \ln \left(\frac{1+x}{1-x} \right)$$

$$21 \quad \checkmark \quad y' = \frac{5(x+1)^4 \cdot (x-1)}{2x^3 \cdot \sqrt{x}}$$

$$22 \quad \checkmark \quad y' = \operatorname{arctg} x$$

$$23 \quad \checkmark \quad y' = \frac{1}{\cos x}$$

$$24 \quad \checkmark \quad y' = \frac{\sqrt{16-x^2}}{4}$$

$$25 \quad \checkmark \quad y' = \frac{10-x}{\sqrt{16x-x^2}}$$

$$26 \quad \checkmark \quad y' = \frac{x^2+1}{x^4+x^2+1}$$

$$27 \quad \checkmark \quad y' = \arcsin \left(\frac{1}{x} \right)$$

$$28 \quad \checkmark \quad y' = \frac{1}{\sin^3 x}$$

$$29 \quad \checkmark \quad y' = \frac{4}{(x^2+1)^3}$$

$$30 \quad \checkmark \quad y' = \frac{\arcsin x}{x^2}$$

$$31 \quad \checkmark \quad y' = \frac{\operatorname{arctg} x}{x^2}$$

$$32 \quad \checkmark \quad y' = \frac{2x \cdot \operatorname{tg} x + 1}{x^2}$$

$$33 \quad \checkmark \quad y' = \frac{-x}{\sqrt{(x^2+1)^3}}$$

$$34 \quad \checkmark \quad y' = \frac{\ln x + 2}{2 \cdot \sqrt{x}}$$

$$35 \quad \checkmark \quad y' = \frac{6x^3 - 3x^2 - 24x - 52}{((x+3) \cdot (x^2+4))^2}$$

$$36 \quad \checkmark \quad y' = \frac{6x \cdot (2x^6 - 1)}{(1+x^6)^2}$$

$$37 \quad \checkmark \quad y' = \frac{-8x^3}{(x^4-1)^2}$$

$$38 \quad \checkmark \quad y' = \frac{28e^{2x} \cdot (4 + e^{2x})}{(4 - e^{2x})^3}$$

$$39 \quad \checkmark \quad y' = \frac{4e^{2x} \cdot (1 + e^{4x})}{(1 - e^{4x})^2}$$

Zadání**Výsledky**

Vypočtete $f'(4)$ pro funkci

$$40) \quad f(x) = \frac{\sqrt{x}}{1 + 2 \cdot \sqrt{x}}$$

$$40 \quad \checkmark \quad f'(4) = 0,01$$

Vypočtete $f'(0)$ a $f''(0)$ pro funkci

$$41) \quad f(x) = \frac{x}{2} \cdot \sqrt{9 - x^2} \cdot \left(\frac{9}{2}\right) \cdot \arcsin \frac{x}{3}$$

$$41 \quad \checkmark \quad f'(0) = 3, \quad f''(0) = 0$$

Vypočtete $f'(5)$ a $f''(5)$ pro funkci

$$42) \quad f(x) = x^2 - x \cdot \sqrt{x^2 - 9} + \ln(x + \sqrt{x^2 - 9})$$

$$42 \quad \checkmark \quad f'(5) = 0, \quad f''(5) = \frac{1}{8}$$

Ve kterých intervalech je derivace zadané funkce kladná?

$$43) \quad y = \operatorname{tg}^2 x + 2 \cdot \ln(\operatorname{tg} x)$$

$$43 \quad \checkmark \quad x \in \left(2k \cdot \frac{\pi}{2}, (2k + 1) \cdot \frac{\pi}{2}\right)$$

Pro které x je derivace zadané funkce rovna nule?

$$44) \quad y = (4x - 1) \cdot e^{\frac{1}{x}}$$

$$44 \quad \checkmark \quad x = \frac{1}{2}$$

Pro která x je derivace zadané funkce rovna nule?

$$45) \quad y = \sin^3 x - 3 \cdot \sin^2 x + 3 \cdot \sin x$$

$$45 \quad \checkmark \quad x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$$

Pro která x platí $f'(x) = 4$ u zadané funkce?

$$46) \quad y = \frac{4 \cdot \sin x}{1 + \cos x}$$

$$46 \quad \checkmark \quad x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$$