

# APLIKACE URČITÉHO INTEGRÁLU:

Obsah povrchu rotačního tělesa

$$y = \sqrt{3+x}$$

$$x \in (-1, 3)$$

$$S = 2\pi \int_{-1}^3 \sqrt{3+x} \cdot \sqrt{1 + \frac{1}{4(3+x)}} dx = 2\pi \int_{-1}^3 \sqrt{3+x} \cdot \sqrt{\frac{4(3+x)+1}{4(3+x)}} dx =$$

$$(\sqrt{3+x})' = \left(\frac{1}{2\sqrt{3+x}}\right)' = \frac{1}{4(3+x)}$$

$$= 2\pi \int_{-1}^3 \sqrt{3+x} \cdot \frac{\sqrt{13+4x}}{2\sqrt{3+x}} dx =$$

$$= \pi \int_{-1}^3 \sqrt{13+4x} dx = \pi \left| \begin{array}{l|l} \sqrt{13+4x} = t & | x=3 \\ 13+4x = t^2 & | t=5 \\ 4dx = 2t dt & | x=-1 \\ dx = \frac{t}{2} dt & | t=3 \end{array} \right| = \pi \int_3^5 t \cdot \frac{t}{2} dt = \frac{\pi}{2} \int_3^5 t^2 dt =$$

$$= \frac{\pi}{2} \left[ \frac{t^3}{3} \right]_3^5 = \frac{\pi}{2} \left( \frac{125}{3} - \frac{27}{3} \right) = \frac{\pi}{2} \left( \frac{98}{3} \right) = \frac{49\pi}{3} \text{ čtvercových jednotek}$$