APLIKACE URČITÉHO INTEGRÁLU:

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$$f(x) = \frac{x^2}{4} - \frac{\ln x}{2} \qquad \text{na intervalu } \langle 1, e\rangle$$

$$l = \int_1^e \sqrt{1 + \frac{(x^2-1)^2}{4x^2}}\,dx = \int_1^e \sqrt{\frac{4x^2 + x^4 - 2x^2 + 1}{4x^2}}\,dx = \int_1^e \sqrt{\frac{x^4 + 2x^2 + 1}{4x^2}}\,dx =$$

$$f'(x) = \frac{1}{\cancel{4}} \cdot \cancel{2}x - \frac{1}{2} \cdot \frac{1}{x} = \frac{x}{2} - \frac{1}{2x} = \frac{x^2-1}{2x}$$

$$(f'(x))^2 = \left(\frac{x^2-1}{2x}\right)^2 = \frac{(x^2-1)^2}{4x^2}$$

$$= \int_1^e \sqrt{\frac{(x^2+1)^2}{4x^2}}\,dx = \int_1^e \frac{x^2+1}{2x}\,dx = \int_1^e \frac{x^{\cancel{2}}}{2\cancel{x}}\,dx + \int_1^e \frac{dx}{2x} =$$

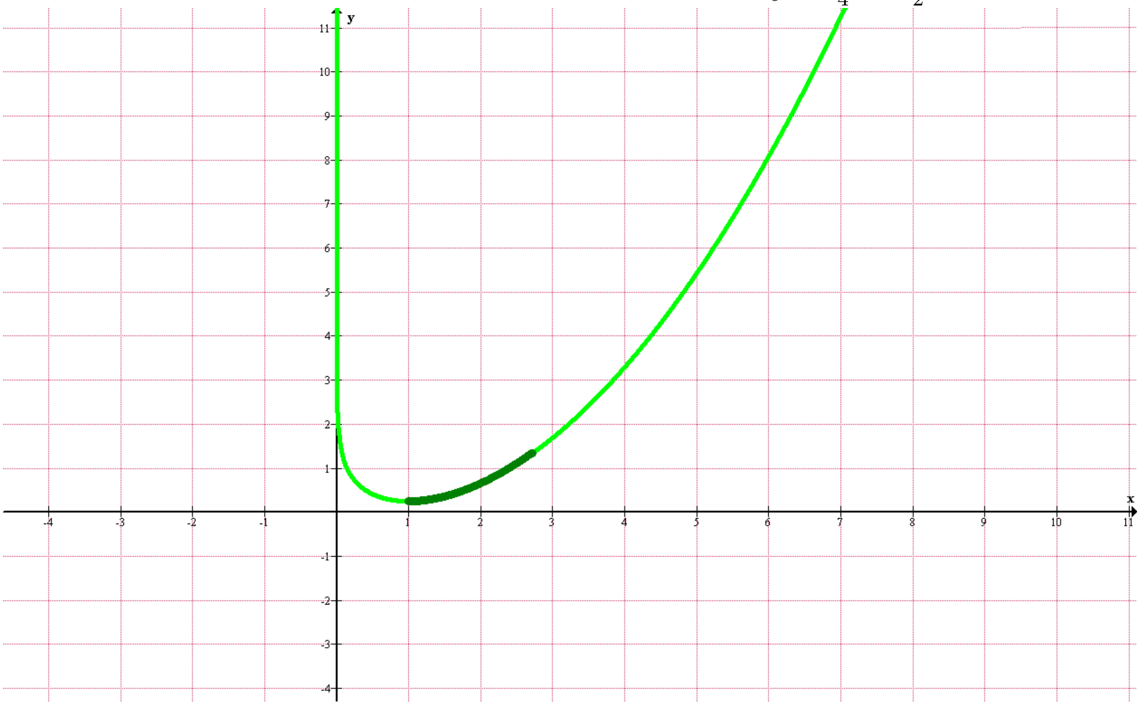
$$= \frac{1}{2}\int_1^e x\,dx + \frac{1}{2}\int_1^e \frac{1}{x}\,dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_1^e + \frac{1}{2}\left[\ln x\right]_1^e =$$

$$= \frac{1}{2}\left[\frac{e^2}{2} - \frac{1}{2}\right] + \frac{1}{2}\left[\ln e - \ln 1\right] = \frac{1}{2}\left[\frac{e^2-1}{2}\right] + \frac{1}{2} \cdot 1 =$$

$$= \frac{1}{2}\left[\frac{e^2-1}{2} + 1\right] = \frac{1}{2} \cdot \frac{e^2+1}{2} = \boxed{\frac{e^2+1}{4}}$$

10b ✓

OBRÁZEK 1. Grafické znázornění funkce $y = \frac{x^2}{4} - \frac{\ln x}{2}$



Zdroj: program Graph