APLIKACE URČITÉHO INTEGRÁLU:

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$$f(x)=\sqrt{9-x^2} \qquad x\in\langle 0;\tfrac{\pi}{2}\rangle$$

$$l=\int_0^{\frac{\pi}{2}}\sqrt{1+\frac{x^2}{9-x^2}}\,dx=\int_0^{\frac{\pi}{2}}\sqrt{\frac{9-x^2+x^2}{9-x^2}}\,dx=\int_0^{\frac{\pi}{2}}\sqrt{\frac{9}{9-x^2}}\,dx=\int_0^{\frac{\pi}{2}}\frac{3}{\sqrt{9-x^2}}\,dx=$$

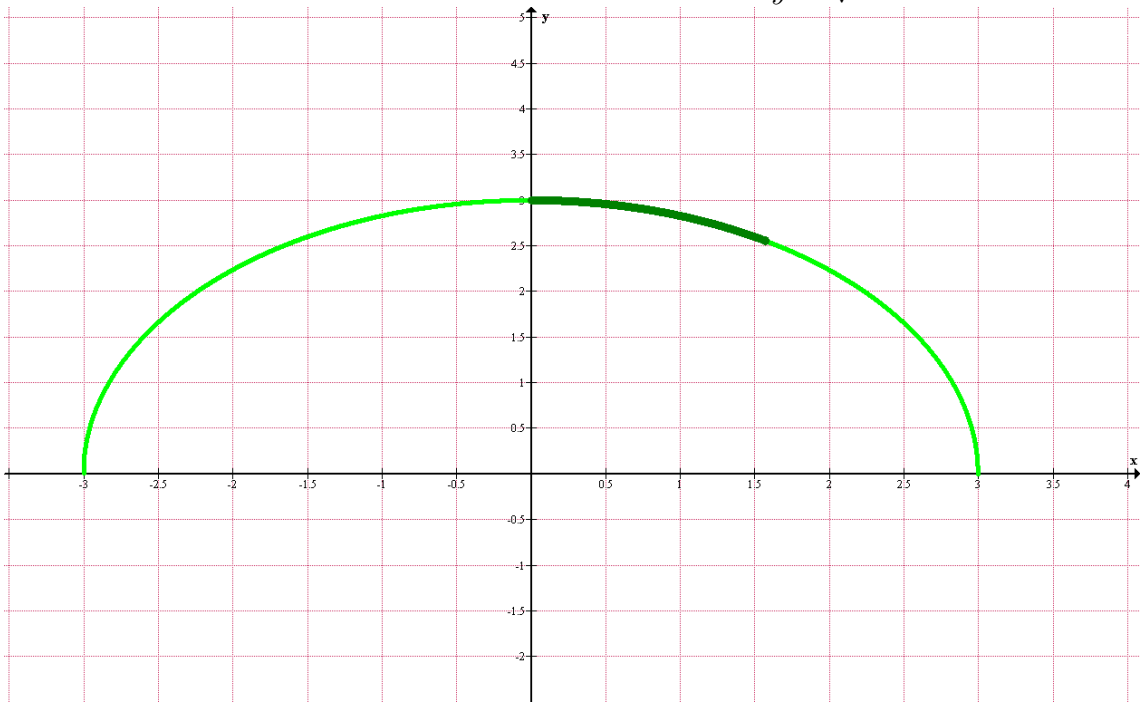
$$f'(x)=\frac{1}{2\sqrt{9-x^2}}\cdot(-2x)=\left(\frac{-x}{\sqrt{9-x^2}}\right)^2=\frac{x^2}{9-x^2}$$

$$=3\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{9-x^2}}\,dx=3\left|\begin{array}{l}x^2=9t^2\ /\sqrt{\ }\\ \boxed{x=3t}\\ dx=3dt\end{array}\right|\begin{array}{l}\text{změna mezí:}\\ x=0\ldots t=0\\ x=\frac{\pi}{2}\ldots t=\frac{\pi}{6}\end{array}\left|=3\int_0^{\frac{\pi}{6}}\frac{1}{\sqrt{9-9t^2}}\cdot 3\,dt=\right.$$

$$=9\int_0^{\frac{\pi}{6}}\frac{1}{\sqrt{9(1-t^2)}}\,dt=9\int_0^{\frac{\pi}{6}}\frac{dt}{3\sqrt{1-t^2}}=3\int_0^{\frac{\pi}{6}}\frac{dt}{\sqrt{1-t^2}}=3\left[\arcsin\tfrac{\pi}{6}-\arcsin 0\right]=$$

$$=3\left(\arcsin\tfrac{\pi}{6}-0\right)=\underline{3\arcsin\tfrac{\pi}{6}}$$ délkových je chuť

Obrázek 1. Grafické znázornění funkce $y = \sqrt{9 - x^2}$



Zdroj: program Graph